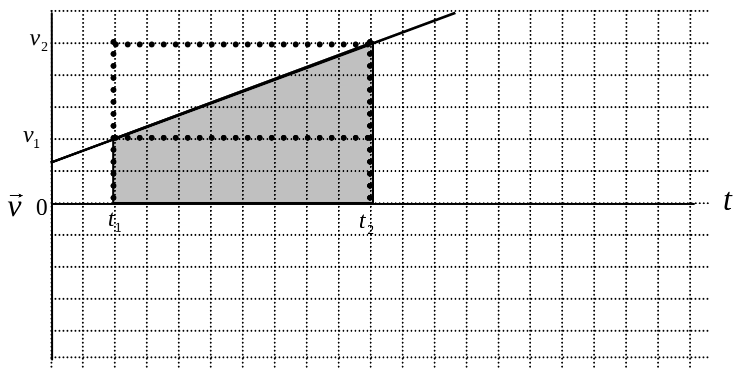
Deriving formulae: We are going to attempt to derive some general formula by analysing the area beneath the following curve. Using 3 methods. Find the area of the shade region by.

a. Finding the sum of the area of the rectangle and triangle of the shade region
b. Finding the area of the trapezoid ($A = \frac{1}{2}(a+b)l$)
c. Finding the area of the large rectangle and subtracting the un-shaded triangle.



a. Finding the displacement by calculating the area of the triangle and rectangle beneath the curve
Area represents $\Delta\vec{d}$

$$\because A = A_{rectangle} + A_{triangle}$$

$$\therefore \Delta\vec{d} = lw + \frac{1}{2}bh$$

$$\Delta\vec{d} = (v_1 - 0)(t_2 - t_1) + \frac{1}{2}(v_2 - v_1)(t_2 - t_1)$$

$$\Delta\vec{d} = (v_1)(\Delta t) + \frac{1}{2}(\Delta v)(t\Delta) \quad but\ we\ know\ \vec{a} = \frac{\Delta v}{\Delta t}\ or\ \Delta v = \vec{a}\Delta t$$

$$\Delta\vec{d} = (v_1)(\Delta t) + \frac{1}{2}(\vec{a}\Delta t)(\Delta t)$$

$$\Delta\vec{d} = (v_1)(\Delta t) + \frac{1}{2}(\vec{a})(\Delta t)^2$$

$$\boxed{\begin{gathered}\Delta\vec{d} = \vec{v}_1(\Delta t) + \tfrac{1}{2}\vec{a}(\Delta t)^2 \\ or \\ \vec{d} = \vec{v}_1 t + \tfrac{1}{2}\vec{a}t^2\end{gathered}}$$

Where $\vec{d}$ is the change in displacement $\vec{v}_1$ is the initial velocity, t is the time interval, and $\vec{a}$ is the acceleration

b. Finding the displacement by calculating the area of trapezoid beneath the curve

Area represents $\Delta\vec{d}$

$$\therefore A = \tfrac{1}{2}(a+b)l$$
$$\therefore \Delta\vec{d} = \tfrac{1}{2}(a+b)l$$
$$\Delta\vec{d} = \tfrac{1}{2}(v_1+v_2)(t_2-t_1)$$
$$\Delta\vec{d} = \tfrac{1}{2}(v_1+v_2)(\Delta t)$$
$$\Delta\vec{d} = \frac{(v_1+v_2)}{2}(\Delta t)$$

$$\boxed{\begin{gathered}\Delta\vec{d} = \frac{(\vec{v}_1+\vec{v}_2)}{2}(\Delta t)\\ or\\ \vec{d} = \frac{(\vec{v}_1+\vec{v}_2)}{2}(t)\end{gathered}}$$

Where $\vec{d}$ is the change in displacement $\vec{v}_1$ is the initial velocity and $\vec{v}_2$ is the final velocity, t is the time interval, and $\vec{a}$ is the acceleration.

c. Finding the displacement by calculating the difference between the area of the large rectangle and the small un-shaded triangle above the curve.

Area represents $\Delta\vec{d}$

$$\because A = A_{rec\tan gle} - A_{triangle}$$
$$\therefore \Delta\vec{d} = lw - \tfrac{1}{2}bh$$
$$\Delta\vec{d} = (v_2-0)(t_2-t_1) - \tfrac{1}{2}(v_2-v_1)(t_2-t_1)$$
$$\Delta\vec{d} = (v_2)(\Delta t) - \tfrac{1}{2}(\Delta v)(t\Delta) \quad but\ \ we\ \ know\, \vec{a} = \frac{\Delta v}{\Delta t}\ \ or\ \ \Delta v = \vec{a}\Delta t$$
$$\Delta\vec{d} = (v_2)(\Delta t) - \tfrac{1}{2}(\vec{a}\Delta t)(\Delta t)$$
$$\Delta\vec{d} = (v_2)(\Delta t) - \tfrac{1}{2}(\vec{a})(\Delta t)^2$$

$$\boxed{\begin{gathered}\Delta\vec{d} = \vec{v}_2(\Delta t) - \tfrac{1}{2}\vec{a}(\Delta t)^2\\ or\\ \vec{d} = \vec{v}_2 t - \tfrac{1}{2}\vec{a}t^2\end{gathered}}$$

Where $\vec{d}$ is the change in displacement $\vec{v}_1$ is the initial velocity and $\vec{v}_2$ is the final velocity, t is the time interval, and $\vec{a}$ is the acceleration.

Important things to note:

i. This equation only **works** for **motion** that is **collinear**. In other words, the velocity, the **displacement** and the **acceleration** must all be occurring on the **same line**. These equations do not work if there is any turning.

ii. These equations work for **distance** and **speed** as well, but again the acceleration and the speed have to be **acting in the same line**

iii. The **acceleration** must be **constant**

Exercise: Using the above formulae, derive the following formula $\vec{v}_2{}^2 = \vec{v}_1{}^2 + 2\vec{a}\vec{d}$

Deriving $\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\vec{d}$

Using the following two equations

$d = \left(\frac{v_1 + v_2}{2}\right)\Delta t$ ① and $a = \frac{\Delta v}{\Delta t}$ ②

rearrange ② to isolate for Δt → $\Delta t = \frac{\Delta v}{a}$

Sub ② into ①

$$d = \left(\frac{v_1 + v_2}{2}\right)\Delta t$$
$$d = \left(\frac{v_1 + v_2}{2}\right)\frac{\Delta v}{a}$$
$$d = \frac{(v_1 + v_2)\Delta v}{2a}$$
$$2ad = (v_1 + v_2)\Delta v$$
$$2ad = (v_1 + v_2)(v_2 - v_1)$$
$$2ad = (v_2 + v_1)(v_2 - v_1) \quad \textit{difference of squares}$$
$$2ad = (v_2)^2 - (v_1)^2$$
$$(v_1)^2 + 2ad = (v_2)^2$$

$$v_2^{\,2} = v_1^{\,2} + 2ad$$
or
$$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\vec{d}$$

Summary of Formulae

Velocity, Displacement, Acceleration

Constant motion (no acceleration)

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$
$$\Delta \vec{d} = \vec{v}\Delta t$$

Constant acceleration

$$\vec{d} = \vec{v}_1 t + \tfrac{1}{2}\vec{a}t^2$$
$$\vec{d} = \frac{(\vec{v}_1 + \vec{v}_2)}{2}(t)$$
$$\vec{d} = \vec{v}_2 t - \tfrac{1}{2}\vec{a}t^2$$
$$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\vec{d}$$
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$
$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$$

Speed, Distance, Acceleration

Constant motion (no acceleration)

$$v = \frac{\Delta d}{\Delta t}$$
$$\Delta d = v\Delta t$$

Constant acceleration

$$d = v_1 t + \tfrac{1}{2}at^2$$
$$d = \frac{(v_1 + v_2)}{2}(t)$$
$$d = v_2 t - \tfrac{1}{2}at^2$$
$$v_2^{\,2} = v_1^{\,2} + 2ad$$
$$a = \frac{\Delta v}{\Delta t}$$
$$v_2 = v_1 + a\Delta t$$