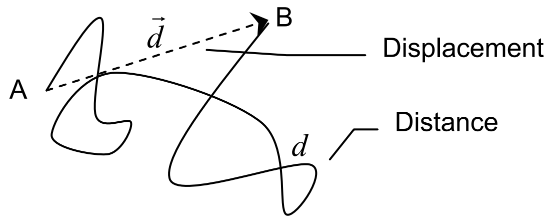
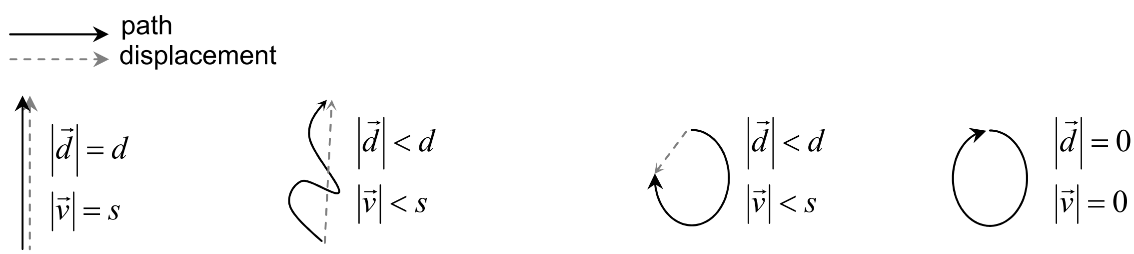
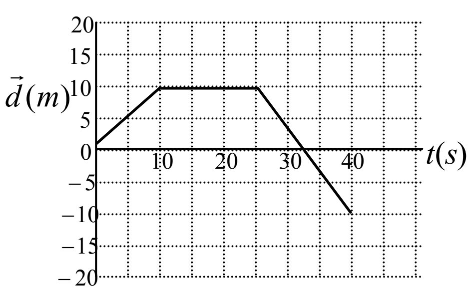
**Displacement:** The straight-line distance between an object's starting position and finishing position, measure as a vector (i.e. includes direction). Displacement is **independent of the path** taken by the object.




**Velocity:** Defined as the object displacement over time. This implies that an object's velocity is often different that the object's speed. Take for instance the example above. The length of the path is the distance ( $d$ ) is much greater than the magnitude of the displacement ( $|\vec{d}|$ ). Therefore the speed would be much greater than the magnitude of the velocity since both occur over the same time interval.




**Interpreting Position vs. Time graphs**




**0s-10s:** Displacement is increasing linearly
Velocity is constant and positive

**10s-25s:** Displacement is constant at 10m
Velocity is zero (at rest)

**25s-32s:** Displacement is decreasing linearly but displacement is still positive.
Velocity is negative

**32s-40s:** Displacement is decreasing linearly and displacement is now negative.
Velocity is still negative

**0s-32s:** Total displacement is 0m, average velocity is 0m/s

**Slope of a Position time Graph**

Taking the slope along sections of a position graph will give you the average velocity over the interval.

**Proof**:
Let $m$ represent the slope along any given section along the graph

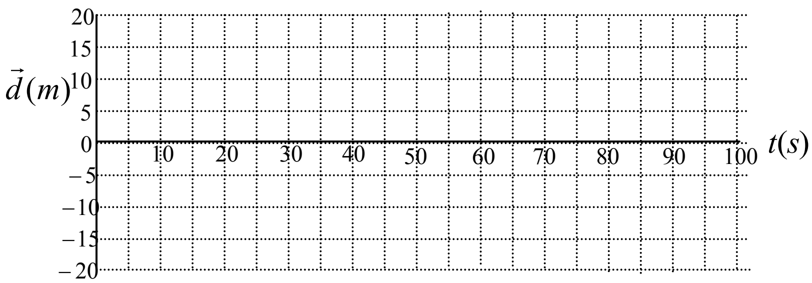
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{\Delta y}{\Delta x}$$
$$m = \frac{\Delta \vec{d}}{\Delta t}$$

$$\because \vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$
$$\therefore m = \vec{v}$$

**Assignment:**

On the position verses time graph below, plot the following positions and times

| Time | Position |
|---|---|
| 0 | 0m |
| 10 | 15m |
| 30 | 15m |
| 45 | 0m |
| 60 | -15m |
| 80 | -15m |
| 90 | 0m |

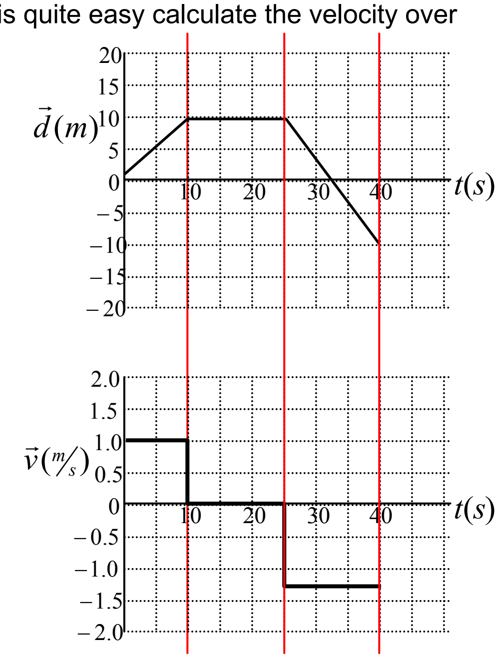



a) Find the displacement and average velocity for the following time intervals
   0-10
   10-20
   30-45
   80-90
b) Find the displacement and average velocity for the following time intervals
   0-10
   0-30
   0-45
   0-80
   0-90
c) During which intervals are the velocity and displacement 0?
d) Predict what the velocity and displacement would be at the 100s mark.

## Interpreting Velocity Versus Time Graphs

Consider the position vs. time graph below. It has been demonstrated earlier that we can use slope to find the average velocity over an interval. Furthermore, we can find the instantaneous velocity using the slope of a tangent at any point along the position versus time graph. In the example below, it is quite easy calculate the velocity over the 3 intervals

**0s-10s:** $\vec{v} = 1.0 m/s$
**10s-25s:** $\vec{v} = 0 m/s$
**25s-40s:** $\vec{v} = -1.33 m/s$




Now compare the $\vec{d}$ vs. $t$ graph with the $\vec{v}$ vs. $t$

The two graphs are lined up along the **t** axis. If you inspect the graph closely, you can see how the d vs. t graph would physically impossible. The change of velocity is instantaneous, which is impossible; there is always some amount of acceleration.

To see how acceleration would be represented on a $\vec{d}$ vs. $t$, we will start with a plausible* $\vec{v}$ vs. $t$ graph. (*means reasonably possible)

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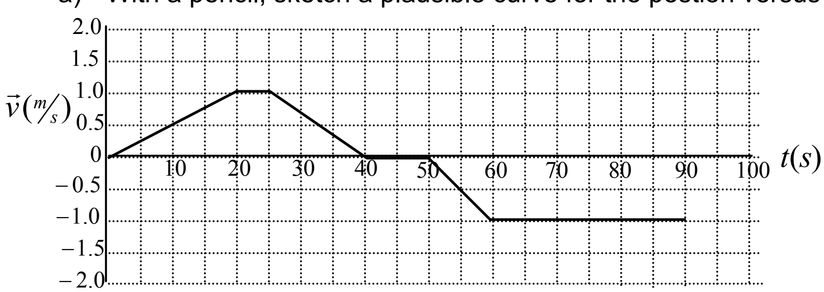
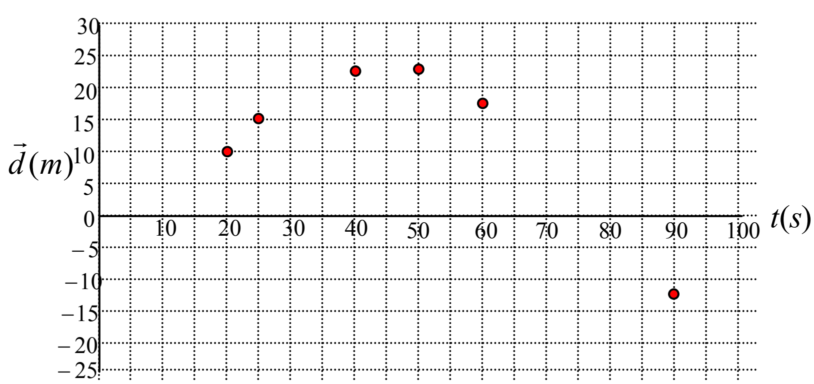
**Exercise:** The following $\vec{v}$ vs. $t$ graph corresponds to the $\vec{d}$ vs. $t$ graph below it. On the $\vec{d}$ vs. $t$ graph, only the position of the object, at certain times, is shown.

a) With a pencil, sketch a plausible curve for the postion versus time graph.

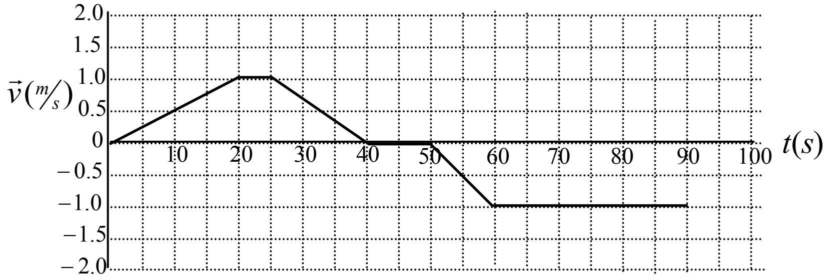


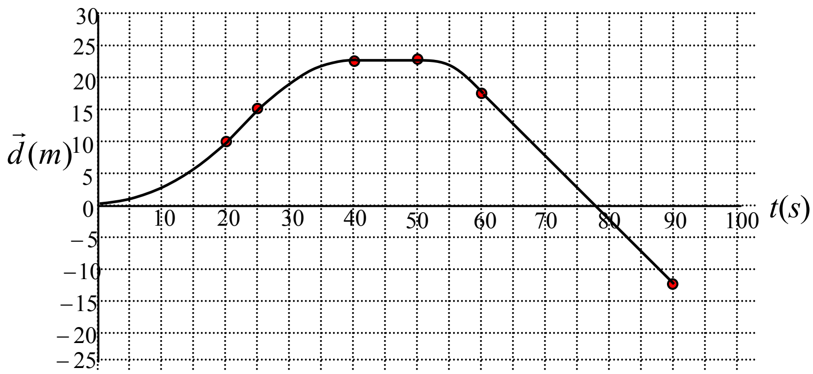




b) On the $\vec{v}$ vs. $t$ graph, calculate the area between the curve and the $t$ axis for the following intervals 0s-20s, 20s-25s, 25s-40s, 40s-50s, 50s-60s, 60s-90s
c) Compare your results in b) to the points on the v vs. t graph
d) How does the slope in the various intervals on the $\vec{v}$ vs. $t$ graph compare to the shape of the curve on the $\vec{d}$ vs. $t$?

**Solution**

a) With a pencil, sketch a plausible curve for the position versus time graph.




b) On the $\vec{v}$ vs. $t$ graph, calculate the area between the curve and the $t$ axis for the following intervals

0s-20s: $A = \frac{1}{2}bh \rightarrow \frac{1}{2}(20)(1.0) = 10m$ Cumulative displacement → 10m

20s-25s: $A = bh \rightarrow (5)(1.0) = 5m$ Cumulative displacement → 15m

25s-40s: $A = \frac{1}{2}bh \rightarrow \frac{1}{2}(15)(1.0) = 7.5m$ Cumulative displacement → 22.5m

40s-50s: $A = bh \rightarrow (10)(0) = 0m$ Cumulative displacement → 22.5m

50s-60s: $A = \frac{1}{2}bh \rightarrow \frac{1}{2}(10)(-1.0) = -5m$ Cumulative displacement → 17.5m

60s-90s: $A = bh \rightarrow (30)(-1.0) = -30m$ Cumulative displacement → -12.5m

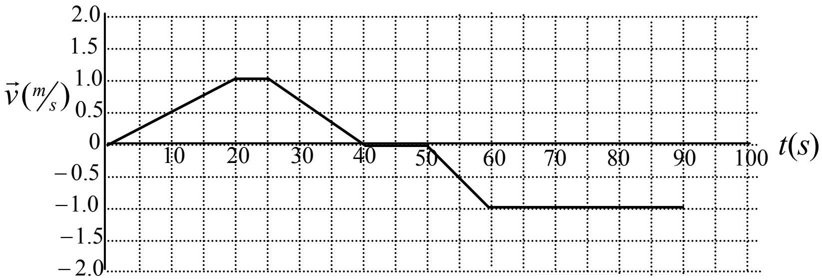
c) Compare your results in b) to the points on the v vs. t graph

0s-20s: Cumulative displacement → 10m
20s-25s: Cumulative displacement → 15m
25s-40s: Cumulative displacement → 22.5m
40s-50s: Cumulative displacement → 22.5m
50s-60s: Cumulative displacement → 17.5m
60s-90s: Cumulative displacement → -12.5m

d) How does the slope in the various intervals on the $\vec{v}$ vs. $t$ graph compare to the shape of the curve on the $\vec{d}$ vs. $t$?

| **Interval** | $\vec{v}$ **vs.** $t$ | $\vec{d}$ **vs.** $t$ **?** |
|---|---|---|
| 0s-20s: | Linear, slope up to the right (+ve) | Non-linear: curves up to the right: slope increasing over the interval (+ve) |
| 20s-25s: | Linear, slope = 0 | Linear: slope is constant, up and to the right |
| 25s-40s: | Linear, Slopes down to the right (-ve) | Non-linear: curves up to the right: but slope is decreasing over the interval (-ve). |
| 40s-50s: | Linear, slope = 0 | Linear: Slope =0 |
| 50s-60s: | Linear, Slopes down to the right (-ve) | Non-linear: curves down to the right, slope is decreasing over the interval (-ve). |
| 60s-90s: | Linear, slope = 0 | Linear: slope is constant, down and to the right |

**Additional analysis:**




On the $\vec{v}$ vs. $t$ graph, calculate

a) the slope of the curve for the following intervals
   0s-20s, 20s-25s, 25s-40s, 40s-50s, 50s-60s, 60s-90s

b) What are the units of the slope?

c) **Deriving formulae:** We are going to attempt to derive some general formula by analysing the area beneath the following curve. Using 3 methods. Find the area of the shade region by.
   a. Finding the sum of the area of the rectangle and triangle of the shade region
   b. Finding the area of the trapezoid ( $A = \frac{1}{2}(a+b)l$ )
   c. Finding the area of the large rectangle and subtracting the un-shaded triangle.