Find the displacement of a boat that sails $1.0km[N30^0W]$, $2.0km[W20^0S]$ and $2.0km[S10^0E]$
Draw a detail diagram.

Solving this question using sine and cosine law is very difficult because the vector diagram is not a triangle. Here is the solution if you were to use sine and cosine law.

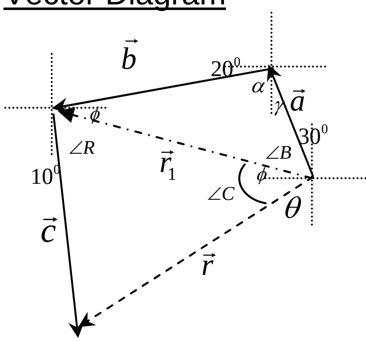
Given	**RTF**	**Formulae**
$\vec{a} = 1.0km[N30^0W]$ $\vec{b} = 2.0km[W20^0S]$ $\vec{c} = 2.0km[S10^0E]$	$\vec{d}$	$r^2 = a^2 + b^2 - 2ab\cos(R)$ $\frac{\sin R}{r} = \frac{\sin B}{b}$ $\frac{\sin C}{c} = \frac{\sin R}{r}$

Solution

Vector Equation

$\vec{r} = \vec{a} + \vec{b} + \vec{c}$

Vector Diagram



First find $\vec{a} + \vec{b}$

Let $\vec{r}_1 = \vec{a} + \vec{b}$

$\gamma = 30^0$ (Z pattern)

$\alpha = 70^0$ (Complementary Angles)

$R_1 = \gamma + \alpha$

$R_1 = 30^0 + 70$

$R_1 = 100^0$

Find $|\vec{r}_1|$

$r_1^2 = a^2 + b^2 - 2ab\cos(R_1)$

$r_1 = \sqrt{a^2 + b^2 - 2ab\cos(R_1)}$

$r_1 = \sqrt{(1.0)^2 + (2.0)^2 - 2(1.0)(2.0)\cos(100)}$

$r_1 = 2.4km$

Find ϕ

$\frac{\sin B}{b} = \frac{\sin R_1}{r_1}$

$\sin B = \frac{b \sin R_1}{r_1}$

$\sin B = \frac{(2.0)\sin(100)}{2.4}$

$\sin B = 0.820673127$

$\angle B = 55.1^0$

$\phi + \angle B + 30^0 = 90^0$

$\phi = 90^0 - \angle B - 30^0$

$\phi = 90^0 - 55.1^0 - 30^0$

$\phi = 4.9^0$

Find $\angle R$

$\phi + \angle R + 10^0 = 90^0$

$\angle R = 90^0 - 10^0 - \phi$

$\angle R = 90^0 - 10^0 - 4.9^0$

$\angle R = 75.1^0$

Find $|\vec{r}|$

$r^2 = c^2 + r_1^2 - 2cr_1\cos(R)$

$r = \sqrt{c^2 + r_1^2 - 2cr_1\cos(R)}$

$r = \sqrt{(2.0)^2 + (2.4)^2 - 2(2.0)(2.4)\cos(75.1^0)}$

$r = 2.70km$

Find θ

$\frac{\sin C}{c} = \frac{\sin R}{r}$

$\sin C = \frac{c \sin R}{r}$

$\sin C = \frac{(2.0)\sin(75.1)}{2.70}$

$\angle C = 45.7^0$

$\theta + \angle C + \angle B + 30^0 = 180^0$

$\theta = 180^0 - \angle C - \angle B - 30^0$

$\theta = 180^0 - 45.7^0 - 55.1 - 30^0$

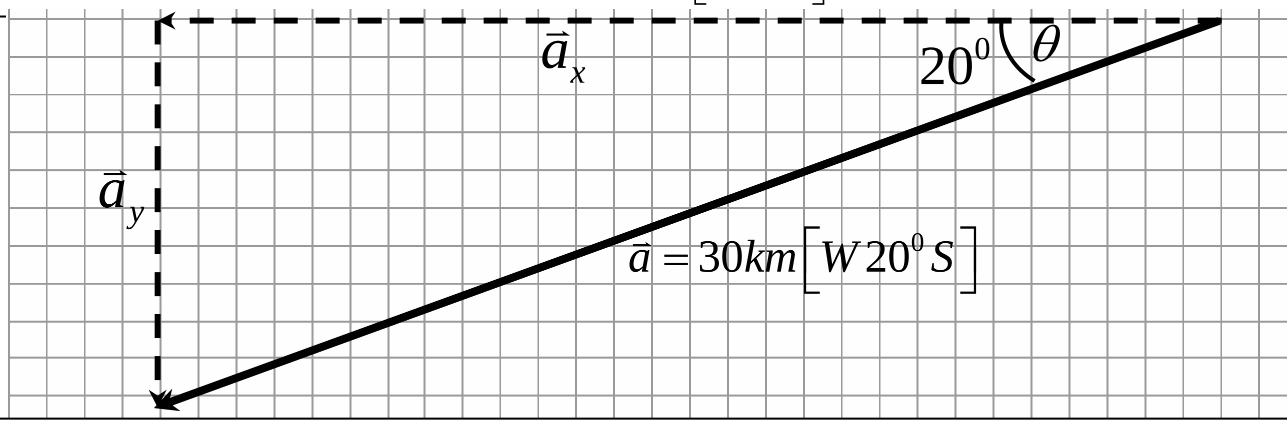
$\theta = 49^0$

$\vec{r} = 2.70km[S49^0W]$

$\therefore \vec{d} = 2.70km[S49^0W]$

*The alternate approach is to use **the component method**. The component method works by breaking your vectors up into their x and y components. For example take the following vector as an example.*

$$\vec{a} = 30km\left[W20^0 S\right]$$



*Vector $\vec{a}$ can be broken up in to two separate vectors, $\vec{a}_x$ and $\vec{a}_y$. These two vectors can replace vector $\vec{a}$ because these two vectors **add** to make vector $\vec{a}$. That is to say that you can get to the same location by traveling the direct route or by traveling along the x then the y vector.*

*To find the length of vector $\vec{a}_x$, we can use **sohcahtoa** specifically $\cos\theta = \frac{a_x}{a}$ because $\vec{a}_x$ is the adjacent side and $\vec{a}$ is the hypotenuse. Rearranging:*

$$\cos(\theta) = \frac{a_x}{a}$$
$$a\cos(\theta) = a_x$$
$$a_x = a\cos(\theta)$$
$$a_x = 30\cos\left(20^0\right)$$
$$\vec{a}_x = 28.19\,\mathrm{km}\left[\mathrm{W}\right]$$

*To find the length of vector $\vec{a}_y$, we can use **sohcahtoa** specifically $\sin\theta = \frac{a_y}{a}$ because $\vec{a}_y$ is the opposite side and $\vec{a}$ is the hypotenuse. Rearranging:*

$$\sin(\theta) = \frac{a_y}{a}$$
$$a\sin(\theta) = a_y$$
$$a_y = a\sin(\theta)$$
$$a_y = 30\sin\left(20^0\right)$$
$$\vec{a}_y = 10.26\,\mathrm{km}\left[\mathrm{S}\right]$$

If you apply Pythagorean theorem, you can see that the two vectors, $\vec{a}_x$ and $\vec{a}_y$ add up to $\vec{a}$

$$a = \sqrt{\left(a_x\right)^2 + \left(a_y\right)^2}$$
$$a = \sqrt{(28.19)^2 + (10.26)^2}$$
$$a = 30km$$

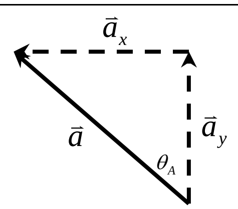
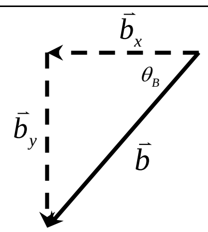
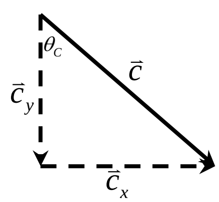
To solve our question:

Find the displacement of a boat that sails $1.0km[N30^0W]$, $2.0km[W20^0S]$ and $2.0km[S10^0E]$
Draw a detail diagram.

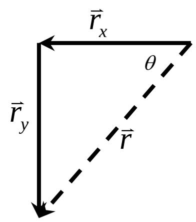
we set up a chart as follows:

Step 1: Create the vector equation: $\vec{r} = \vec{a} + \vec{b} + \vec{c}$

Step 2: Create the following chart. Note: the directions are obvious when you look at the vectors. For example, the first vector is $1.0km[N30^0W]$*. If you examine the diagram, the first vector moves N than W, the same order they appear above. Also in the chart, you can see that the x-comp is moving [W] and the y-component is moving [N]*

Vector	**x-comp**	**y-comp**
	$a_x = a\sin(\theta_A)$ $a_x = 1.0\sin(30^0)$ $\vec{a}_x = 0.5\,km[W]$	$a_y = a\cos(\theta_A)$ $a_y = 1.0\cos(30^0)$ $\vec{a}_y = 0.866\,\text{km}[\text{N}]$
	$b_x = b\cos(\theta_B)$ $b_x = 2.0\cos(20^0)$ $\vec{b}_x = 1.879\ km[W]$	$b_y = b\sin(\theta_B)$ $b_y = 2.0\sin(20^0)$ $\vec{b}_y = 0.684\ km[S]$
	$c_x = c\sin(\theta_C)$ $c_x = 2.0\sin(10^0)$ $\vec{c}_x = 0.3473\ km[E]$	$c_y = c\cos(\theta_C)$ $c_y = 2.0\cos(10^0)$ $\vec{c}_y = 1.97km[S]$
Now that we have all our components we can find our resultants in "x" and "y". Since all the "x" vectors are collinear we can use "+" for [E] and "-" for [W] . The same is true for the "y" vectors. we can use "+" for [N] and "-" for [S]		
	$\vec{r}_x = \vec{a}_x + \vec{b}_x + \vec{c}_x$ $r_x = (-0.5) + (-1.879) + (+0.3473)$ $r_x = -2.032km$ $\vec{r}_x = 2.032km[W]$	$\vec{r}_y = \vec{a}_y + \vec{b}_y + \vec{c}_y$ $\vec{r}_y = (+0.866) + (-0.684) + (-1.97)$ $r_y = -1.788km$ $\vec{r}_y = 1.788km[S]$

Finally, we can find the magnitude of the resultant ($\left|\vec{r}\right|$) by using Pythagorean Theorem and Trigonometry.

<table>
<tr><th>New Vector Equation</th><th>Find $\left|\vec{r}\right|$</th><th>Find θ</th></tr>
<tr><td>$\vec{r} = \vec{r}_x + \vec{r}_y$</td><td rowspan="3">$\left|\vec{r}\right| = \sqrt{\left|r_x\right|^2 + \left|r_y\right|^2}$
$\left|\vec{r}\right| = \sqrt{(2.032)^2 + (1.788)^2}$
$\left|\vec{r}\right| = 2.70\text{km}$</td><td rowspan="3">$\tan\theta = \left(\frac{r_y}{r_x}\right)$
$\theta = \tan^{-1}\left(\frac{1.788}{2.032}\right)$
$\theta = 41.3^0$</td></tr>
<tr><td>New Vector Diagram</td></tr>
<tr><td></td></tr>
<tr><td colspan="3">$\therefore \vec{d} = 2.70km\left[W\,41.3^0\,S\right]$ or $\vec{d} = 2.70km\left[S\,48.7^0\,W\right]$</td></tr>
</table>