Vector addition can be a very tedious task if you don't work intelligently. Often you will be solving questions that have many vectors, many of which will not be collinear. There are some techniques that will make your life easier but does involve a little bit of organization on your part.

Adding or Subtracting Multiple vectors

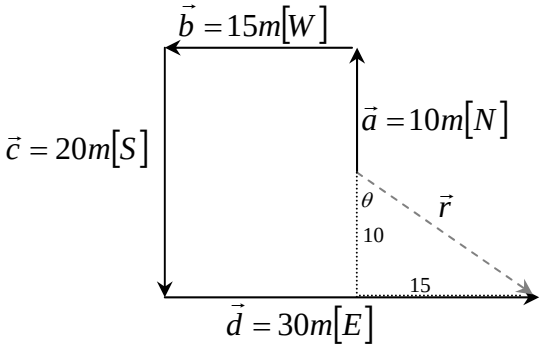
Take for instance the following example.

$\vec{a} = 10m[N]$
$\vec{b} = 15m[W]$
$\vec{c} = 20m[S]$
$\vec{d} = 30m[E]$

Find $\vec{a} + \vec{b} + \vec{c} + \vec{d}$

Method 1- The Geometric Perspective

Draw a fully labelled diagram and analyse visually.



Find $|\vec{r}|$

$|\vec{r}| = \sqrt{x^2 + y^2}$
$r = \sqrt{(15)^2 + (10)^2}$
$r = 18m$

Find θ

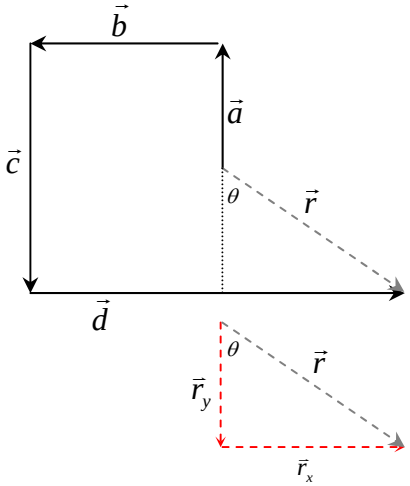
$\tan\theta = \frac{opp}{adj}$
$\tan\theta = \frac{15}{10}$
$\theta = \tan^{-1}\left(\frac{15}{10}\right)$
$\theta = 56^0$

$\therefore \vec{r} = 18m[S56^0E]$

Method 2- The Mathematical Perspective

Draw a labelled diagram, rewrite the vector equation such that the collinear vectors are grouped together, Solve for the resultant for each group of collinear vectors separately and then find the final resultant by adding the two intermediate resultants.

Given
$\vec{a} = 10m[N]$
$\vec{b} = 15m[W]$
$\vec{c} = 20m[S]$
$\vec{d} = 30m[E]$



Find $|\vec{r}|$

$\vec{r} = \vec{a} + \vec{b} + \vec{c} + \vec{d}$
$\vec{r} = \vec{a} + \vec{c} + \vec{b} + \vec{d}$

Let N and E be positive

$\vec{r}_y = \vec{a} + \vec{c}$
$\vec{r}_y = 10 + (-20)$
$\vec{r}_y = -10$
$\vec{r}_y = 10m[S]$

$\vec{r}_x = \vec{b} + \vec{d}$
$\vec{r}_x = (-15) + 30$
$\vec{r}_x = 15$
$\vec{r}_x = 15m[E]$

$|\vec{r}| = \sqrt{|\vec{r}_y|^2 + |\vec{r}_x|^2}$
$r = \sqrt{(10)^2 + (15)^2}$
$r = 18m$

Find θ

$\tan\theta = \frac{opp}{adj}$
$\tan\theta = \frac{15}{10}$
$\theta = \tan^{-1}\left(\frac{15}{10}\right)$
$\theta = 56^0$

$\therefore \vec{r} = 18m[S56^0E]$

Q: When should you use Method 1 and when should you use Method 2?
A: Use what ever is more convenient. Method 1 is fine when you have only a few vectors in each orientation. It becomes less convenient when you have three or more vectors along any given orientation.

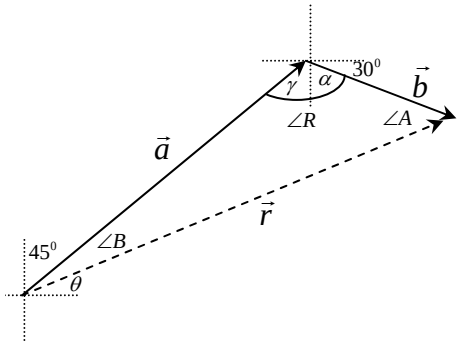
Using Sine Law and Cosine Law for Vector Addition

When adding vectors that are not orthagonal (at right angles to each other), the vector addition becomes a bit more involved. In thes cases you will be required to use sine and cosine law to solve these.

Ex:

$\vec{a} = 20km\left[N45^0E\right]$
$\vec{b} = 10km\left[E30^0S\right]$

Find $\vec{a}+\vec{b}$



- First, draw a detail diagram, as close to scale as possible.
- Add "cross hairs" at the base of each vector in order to help you find unknown angles.
- Label the vectors and the known angles
- Label the unknown angles with appropriate symbols
- Use the various angle thermos to solve for the unknown angles

Find unknown angles

$\gamma = 45^0$ (Z Pattern)
$\alpha = 60^0$ (Complementary Angles)
$\angle R = \alpha + \gamma$
$= 45^0 + 60^0$
$= 105^0$

Find magnitude of the resultant using cosine Law

$$r^2 = a^2 + b^2 - 2ab\cos(R)$$
$$r = \sqrt{a^2 + b^2 - 2ab\cos(R)}$$
$$r = \sqrt{(20)^2 + (10)^2 - 2(20)(10)\cos(105^0)}$$
$$r = 24.6km$$

Find angle *B* using sine law

$$\frac{\sin B}{b} = \frac{\sin R}{r}$$
$$\frac{\sin B}{10} = \frac{\sin(105^0)}{24.6}$$
$$\sin B = \frac{10\cdot\sin(105^0)}{24.6}$$
$$\sin B = 0.392652774$$
$$B = 23.1^0$$

Find angle *θ*

$45^o + \angle B + \theta = 90$
$\theta = 90 - 45 - 23.1$
$\theta = 21.9$

$\therefore \vec{r} = 25km\left[E22^0N\right]$

Assignment

1. John walk 5.00km [W] then 6.00km [N]

   a) Find the distance John covered
   b) Find the displacement

2. Find the displacement of a cruise ship that travels $20.00km\left[N\right]$ and then $60.00km\left[W60.00^{0}S\right]$
   Draw a detail diagram.

3. Find the displacement of a boat that sails $1.00\,km\left[N30.0^{0}W\right]$, $2.00\,km\left[W20.0^{0}S\right]$ and $2.00km\left[S10.0^{0}E\right]$
   Draw a detail diagram.