In physics, we are often concerned with the *direction* of an object's motions. To represent direction we need to use vectors.

Ex: "John was traveling at a **speed** of 100km/h" compared to "John was traveling at a **velocity** of 100km/h [E]

Vector quantities indicate direction.

Scalars

Any measured quantity that makes no reference to direction is called a scalar.

Ex: 30 seconds, 59kg, 100W, and 10m

Two components, a numerical value and a unit

30s

Vectors

Any measured quantity that makes reference to direction

Ex: 30m [Up], 5 m/s [W], 200N [Down]

Three components, a numerical value and a unit and direction

30m [W]

Vector Coordinate System

There are **three** ways to state direction with vectors, two methods use the **compass system** and the other method uses a ***polar coordinate*** system.

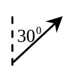
Method 1

This is the most straight forward of the three systems and hence the preferred method of stating direction

Example: $[N30^0 E]$

This means the vector is pointing 30^0 degrees off of North, towards the east. The easiest way to think of it is

1. you point north 

2. rotate 30^0 to the east 

Method 2

This method is not as intuitive as the first. It is also based on the coordinate systems of the compass.

Example: $[30^0 E\, of\, N]$

This also means that the vector points 30^0 degrees off of North, towards the east. Although the direction is stated exactly how you would state it in sentence form, it is not as intuitive to draw as the first method.

Converting from Method 2 to Method 1:

Example: $[30^0 E\, of\, N]$

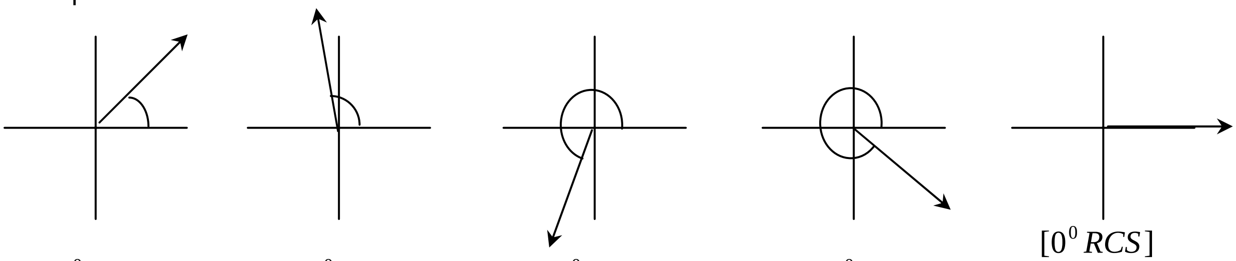
[$30^0 E$ ~~of~~ N]

$[N30^0 E]$

Method 3

This method is called the **rectangular coordinate system** or **RCS** for short. This method uses the **polar coordinate** system, which is used quite extensively in the later years in math and physics. It is based on a direction system that measures the angle between a base line that point east and the vector, measured in the counter clockwise direction.

Example:



$[45^0 RCS]$ $[100^0 RCS]$ $[250^0 RCS]$ $[320^0 RCS]$ $[0^0 RCS]$ *or* $[360^0 RCS]$

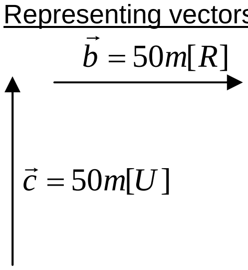
Position and navigation

"Polar coordinates are used often in navigation, as the destination or direction of travel can be given as an angle and distance from the object being considered. For instance, aircraft use a slightly modified version of the polar coordinates for navigation. In this system, the one generally used for any sort of navigation, the 0° ray is generally called heading 360, and the angles continue in a clockwise direction, rather than counterclockwise, as in the mathematical system. Heading 360 corresponds to magnetic north, while headings 90, 180, and 270 correspond to magnetic east, south, and west, respectively.[23] Thus, an aircraft traveling 5 nautical miles due east will be traveling 5 units at heading 90 (read zero-niner-zero by air traffic control)" (**source**: http://en.wikipedia.org/wiki/Polar_coordinate_system#Position_and_navigation)

Comparing traditional measured quantities: Scalar vs. Vector

Scalar Name	Vector Name
Distance (d) → 10m	Displacement ($\vec{d}$)→ $10m[U]$
Speed (s) → 20m/s	Velocity ($\vec{v}$)→ $20m/s[N45^o E]$
Acceleration (a) → $9.8m/s^2$	Acceleration ($\vec{a}$)→ $9.8m/s^2[D]$

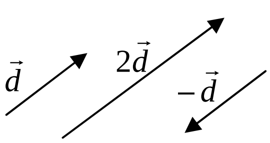
Representing vectors graphically



arrow indicates direction of the vector.

Multiplying Vectors by Scalars

Multiplying vectors by scalars only affect the length of the vector not the direction, unless the vector is multiplied by



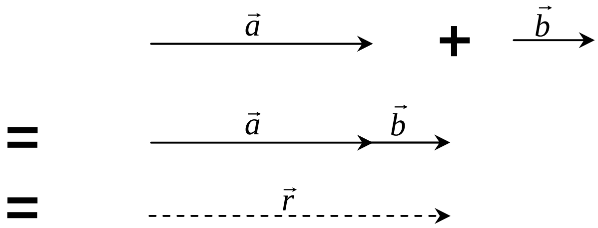
a negative scalar, in which case the direction will reverse.

Adding and Subtracting Vectors

Collinear Vectors (on the same line)

Addition

Ex1:



Align the vectors head to tail.

Mathematically $\vec{r} = \vec{a} + \vec{b}$ ($\vec{r}$ stands for **resultant**)

if $\vec{a} = 50cm[R]$ and $\vec{b} = 20cm[R]$

Let R be positive

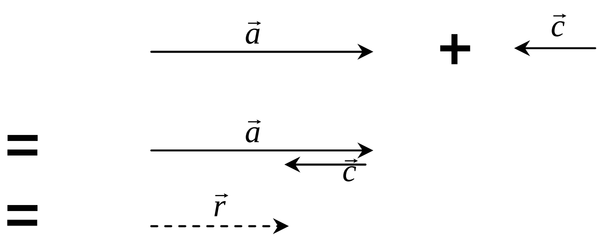
$\vec{r} = \vec{a} + \vec{b}$

$r = 50 + 20$

$r = 70$

$\vec{r} = 70cm[R]$

Ex2:



Align the vectors head to tail

Mathematically $\vec{r} = \vec{a} + \vec{c}$ **(**$\vec{r}$ **stands for** resultant**)**

if $\vec{a} = 50cm[R]$ and $\vec{c} = 20cm[L]$

Let R be positive

$\vec{r} = \vec{a} + \vec{c}$

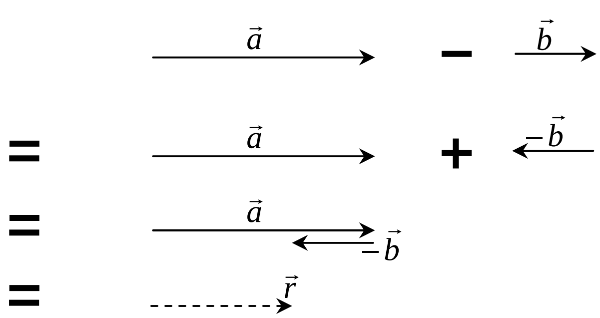
$r = 50 + (-20)$

$r = 30$

$\vec{r} = 30cm[R]$

Subtraction

Ex1:



if $\vec{a} = 50cm[R]$ and $\vec{b} = 20cm[R]$

$\vec{r} = \vec{a} - \vec{b}$

$\vec{r} = \vec{a} + \left(-\vec{b}\right)$

$r = 50 + (-20)$

$r = 30$

$\vec{r} = 30cm[R]$

Combined scalar multiplication and addition

Ex1:

$\vec{a} = 50cm[R]$ and $\vec{b} = 20cm[R]$

Find $5\vec{a} - 2\vec{b}$

Let R be positive, L be negative

$$\vec{r} = 5\vec{a} - 2\vec{b}$$
$$r = 5(50) - 2(20)$$
$$r = 250 - 40$$
$$r = 210$$
$$\vec{r} = 210cm[R]$$

Ex2:

$\vec{a} = 50cm[R]$ and $\vec{c} = 20cm[L]$

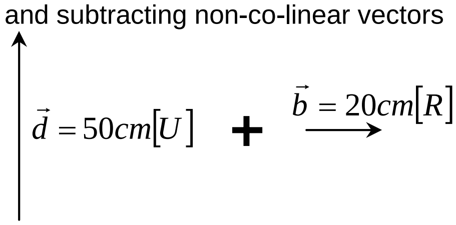
Find $-10\vec{a} - 5\vec{c}$

Let R be positive, L be negative

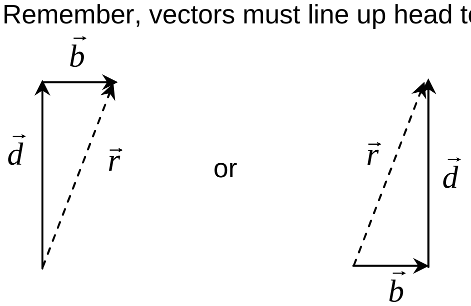
$$\vec{r} = -10\vec{a} - 5\vec{c}$$
$$r = -10(50) - 5(-20)$$
$$r = -500 + 100$$
$$r = -400$$
$$\vec{r} = 400cm[L]$$

Non Co-Linear Vectors

Adding and subtracting non-co-linear vectors



Remember, vectors must line up head to tail



$$\vec{r} = \vec{d} + \vec{b}$$

To find the magnitude of $\vec{r}$, we may use Pythagorean theorem

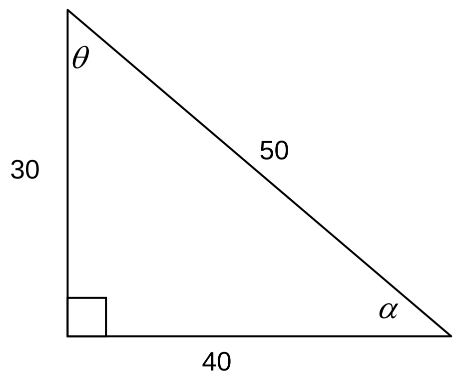
Note: $|\vec{r}| = r$ (the absolute value symbol in physics gives us the magnitude of the vector. This is the same as expressing the vector without the vector symbol above the symbol, i.e. r instead of $\vec{r}$.

$$|\vec{r}| = \sqrt{|\vec{d}|^2 + |\vec{b}|^2}$$
$$r = \sqrt{(d)^2 + (b)^2}$$
$$r = \sqrt{(50)^2 + (20)^2}$$
$$r = 53.9cm$$

$\therefore |\vec{r}| = 54cm$, Note: We have not stated the answer with direction yet. This requires trigonometry, which will be discussed next.

Trigonometry

$\sin\theta = \frac{opp}{hyp}$, $\cos\theta = \frac{adj}{hyp}$, $\tan\theta = \frac{opp}{adj}$ or remember **sohcahtoa.**



$$\sin\theta = \frac{40}{50} \qquad \sin\alpha = \frac{30}{50}$$
$$\cos\theta = \frac{30}{50} \qquad \cos\alpha = \frac{40}{50}$$
$$\tan\theta = \frac{40}{30} \qquad \tan\alpha = \frac{30}{40}$$

Sine Law and Cosine Law

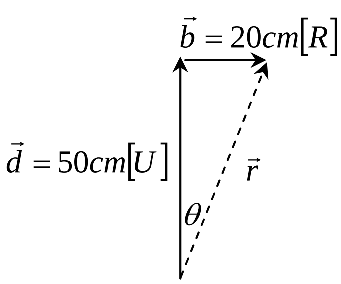
Sine Law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Law

$$c^2 = a^2 + b^2 - 2ab\cos(C) \ or$$
$$a^2 = c^2 + b^2 - 2cb\cos(A) \ or$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

…Continuing from the previous example



Find θ

$$\tan\theta = \frac{opp}{adj}$$
$$\tan\theta = \frac{b}{d}$$
$$\theta = \tan^{-1}\left(\frac{b}{d}\right) \qquad \therefore \vec{r} = 54cm[U\,22^0\,R]$$
$$\theta = \tan^{-1}\left(\frac{20}{50}\right)$$
$$\theta = 22^0$$

Assignment

Using the vectors $\vec{a} = 10m[R]$, $\vec{b} = 20m[L]$, $\vec{c} = 10m[U]$, $\vec{d} = 5m[D]$

1. Draw the following vector diagrams

    a. $\vec{a} + \vec{b}$
    b. $\vec{c} - \vec{d}$
    c. $2\vec{a} - \frac{1}{2}\vec{b}$
    d. $\vec{a} + \vec{c} + \vec{b}$

2. Solve the vector equations in 1) mathematically.

3. Draw and solve the following vector equations

    a. $\vec{a} + 2\vec{c}$
    b. $\vec{b} - \vec{d}$
    c. $2\vec{a} - 4\vec{c} + \vec{b} - 2\vec{d}$