

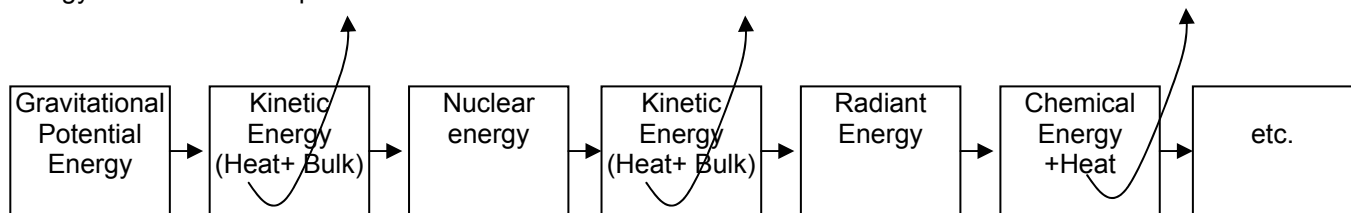
**Energy** is defined as the **ability to do “work”**.

- Energy is **transferred** from **one object to another** when work is done.
- Energy comes in **many forms** that are **interchangeable**
- Energy can be **stored** and used later to do work
- Energy is **always conserved** meaning that it **cannot be created or destroyed**.

**All energy**, at some point, is **converted to heat** and dissipated away. Unfortunately, this heat energy is not in a usable form because it is so widely dispersed away.

For example: **Solar energy**.

Most life on Earth is dependent on the Sun's energy. Sunlight is necessary for *photosynthesis*, a process that creates chemical energy in the form of glucose. Where does the sunlight come from? Sunlight is produced as a result of the tremendous heat generated within the Sun's core. This heat is generated by massive **nuclear reaction** that is drive by the immense pressure generated by the Sun's immense mass (gravity). Consider the energy transformation equation.



### Einstein's Genius

Einstein correctly theorized that matter and energy are two forms of the same thing. That is to say, matter and energy are completely interchangeable. He defined the relationship by quantifying the amount of energy stored in an object at rest due to its mass in the probably the most well know physics equation in History.

$$E = mc^2$$

where  $E$  is the energy in Joules ( $J$ )  
 $m$  is the mass defect in kg  
 $c$  is the speed of light  $3 \times 10^8 \text{ m/s}$

**Q:** So how much energy is stored in every-day matter anyway?

**A:** Well consider the bomb that was dropped on Hiroshima Japan. On that day, an entire city of population 100,000 was destroyed in an instant buy a relatively small nuclear bomb by today's standards. The entire city was levelled and the human death toll was almost total. The amount of mass converted to energy during this horrific event was about 12 grams, about the equivalent mass of a pack of “sour chews”

## Energy - Two Categories

There are two forms of energy. **Potential** and **Kinetic**

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**Potential Energy:** is stored energy due to and object's state. It is called potential energy because this stored energy has the potential to do work.

### Types of potential energy

**Chemical:** Energy stored within the molecules of a substance, i.e. foods and fuel. Energy is released as heat during complex chemical reactions. This released energy can be harnessed to do work.

**Nuclear:** Energy stored within the nucleus of the atom. Tremendous energy is released when the nucleus of an atom is split or fused.

**Gravitational:** The stored energy of an object due to its position above a large celestial body. Energy released from falling objects can do work upon impact.

**Elastic Potential:** Energy stored by bending, stretching or compressing elastic materials. Temporary deformation of an elastic material will always return to its initial state. This tendency can be used to accelerate

objects, thereby imparting the potential energy as kinetic energy.

**Electric Potential:** The stored energy of charged objects due to their relative positions and charge. Like charges cause repulsive forces resulting in the potential acceleration of the objects away from each other where unlike charges cause attractive forces resulting in the potential acceleration of the objects toward each other.

**Magnetic Potential:** The stored energy of magnetized objects due to their relative positions and polarization. Like poles cause repulsive forces resulting in the potential acceleration of the objects away from each other where unlike poles cause attractive forces resulting in the potential acceleration of the objects toward each other.

**Kinetic Energy:** is the energy of motion

### Types of kinetic energy

**Bulk Kinetic:** Energy that is due to motion of the entire object i.e. a rolling cue ball

**Sound:** Energy transmitted from molecule to molecule within a medium in the form of longitudinal sound waves

**Thermal:** Energy an object has due to the random motion of the molecules within a substance

**Radiant:** Energy that is transmitted via electromagnetic waves such as radio, light, x-rays etc.

**Electric:** Energy associated with moving electric charges.

**Work:** Work is defined as a force through a displacement. According to this definition, work is only done if the force acting on the object is in a direct line with the motion of the object. Work is a **scalar** and can be found using the formula

$$W = \vec{F} \cdot \vec{d}$$

Where  $W$  is the work in Joules (J),

$\vec{F}$  is the force in Newtons (N), and

$\vec{d}$  is the displacement in meters (m).

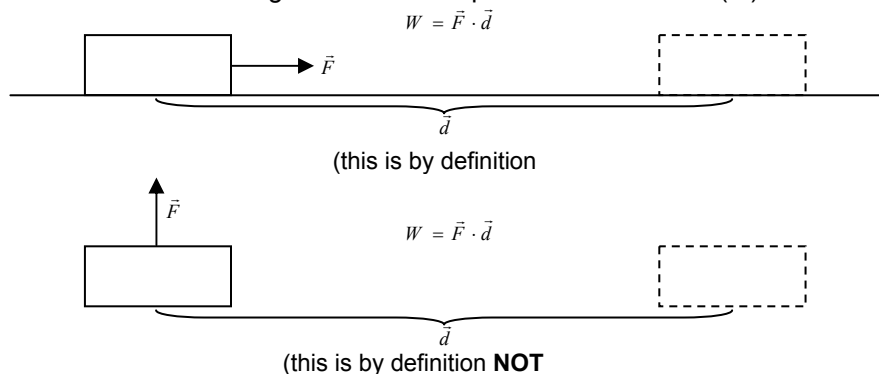
Work does not have direction so this mathematical operation is what is called the **dot product**, which is a form of vector multiplication. The above **formula assumes** that the forces are **collinear**. For our purposes, we will take the product of the magnitudes of our vectors if the force and displacement vectors are **collinear**. If the force and displacement are not collinear, the formula used is

$$W = F \cdot d \cos \theta$$

Where  $W$  is the work in Joules (J),

$F$  is the magnitude of the force in Newtons (N), and

$d$  is the magnitude of the displacement in meters (m).



**Note:** often work is defined as a **force** through a **distance**. Since **displacement** and **distance** are the same in magnitude if the motion is in a straight line, the distinction is not an issue. If however, the force and motion are not collinear, then one must use the displacement explicitly.

**Example:** A 5kg sled is pushed by a net force of 15N. How much work is done on the sled if the sled moves

a) 5m

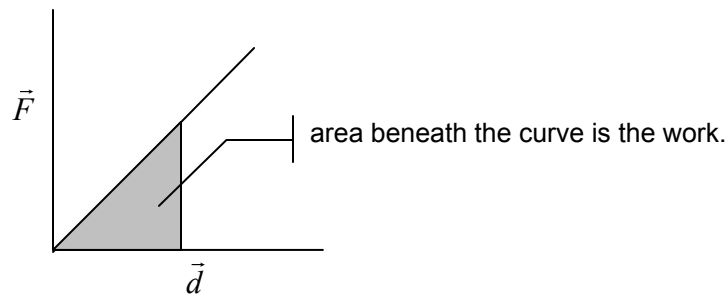
$$\begin{aligned} W &= \vec{F} \cdot \vec{d} \\ &= F \cdot d \\ &= 15N \cdot 5m \\ &= 75N \cdot m \\ &= 75J \end{aligned}$$

b) 10m

$$\begin{aligned} W &= \vec{F} \cdot \vec{d} \\ &= F \cdot d \\ &= 15N \cdot 10m \\ &= 150N \cdot m \\ &= 150J \end{aligned}$$

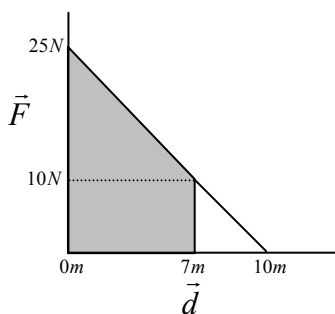
### Work when F is not constant:

Occasionally, one runs into situations where force is not constant over the displacement. If the force can be determined along various points along the displacement, a Force vs. displacement graph can be generated. The area beneath a force vs. displacement graph is the Work.



**Example:** Find the work done on a 500g car that is powered by a spring loaded motor over the first 7m?

Below is a Force vs. Distance plot. This implies that the force produced by the spring motor decreases the further the car travels. According to the graph, the motor will no longer produce a force once the cart has traveled 10m. The greyed out area is the portion of the graph that represents the work done on the car over the first 7m.



Area of the rectangle

$$\begin{aligned} A &= bh \\ &= (7m)(10N) \\ &= 70N \cdot m \\ &= 70J \end{aligned}$$

Area of the triangle

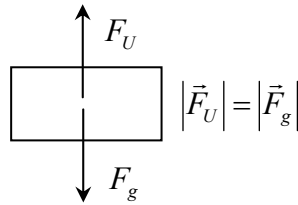
$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(7m)(25N - 10N) \\ &= 52.5N \cdot m \\ &= 52.5J \end{aligned}$$

$\therefore$  Work is 122.5J

**Work and Gravity:** When a force is exerted upward on any mass causing it to increase in height, work is done on the object. That work is stored as gravitational potential energy ( $E_g$ ). From experience, one would know this concept implicitly through everyday household accidents.

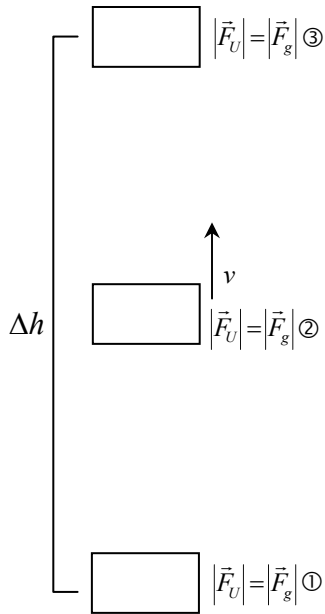
**Ex:** dropping a cup on a counter is less likely to break than if it were to be dropped on the floor. Hence, the greater the height, the harder the fall. More work is done on the object at impact as the height of the object is increased.

Analysing work due to gravity



The minimum force required to raise any object must be at least equal to the weight of the object.

By closely analysing the above FBD, one should notice that  $F_{net} = 0N$ . It may seem odd how motion can occur in this situation since all the forces are balanced. According to Newton's first law, all objects will remain in a state of rest or uniform motion in a straight line unless acted upon by an unbalanced external force. Even though the forces are balanced, the object can continue to move in a straight line, at a constant velocity. But how does it get started? Observe.



- ① Initially,  $F_{net} = 0$ , therefore  $|\vec{F}_U| = |\vec{F}_g|$ . To get the system moving, a slight impulse is required for a short period of time ( $|\vec{F}_U| > |\vec{F}_g|$ ), after words the forces rebalance and the system moves up at a constant velocity ( $|\vec{F}_U| = |\vec{F}_g|$ ).
- ② The system continues to moves up at a **constant** velocity ( $|\vec{F}_U| = |\vec{F}_g|$ ).
- ③ Finally to stop the system, the upward force has to be reduced in order that gravity takes over and decelerates the upward motion ( $|\vec{F}_U| < |\vec{F}_g|$ ), after words the forces rebalance and the system is again at rest ( $|\vec{F}_U| = |\vec{F}_g|$ ).

Without getting too deeply into the analysis, one can see that the average upward force required to move a mass against gravity is the same as the weight of the object itself ( $|\vec{F}_U| = |\vec{F}_g|$ )

$$\begin{aligned}
 W &= |\vec{F}_U| \cdot |\vec{d}| \\
 &= F_U \cdot d \\
 &= F_g \cdot \Delta h \\
 &= mg\Delta h
 \end{aligned}$$

$\therefore W = mg\Delta h \text{ or } W = \Delta E_g$

Where  $W$  is the work in Joules ( $J$ ),  
 $m$  is in mass in  $kg$ , and  
 $\Delta h$  is the change in height in  $m$ .

**Example 1:** How much work is required to raise a 30kg object to a height of 10m?

To answer this, one must define the ground as zero since it is the lowest point and no additional gravitational potential energy ( $E_g$ ) can be lost.

$$\begin{aligned} W &= mg\Delta h \\ &= (30\text{kg})(9.8\text{N/kg})(10\text{m} - 0\text{m}) \\ &= 2940\text{N}\cdot\text{m} \\ &= 2940\text{J} \end{aligned}$$

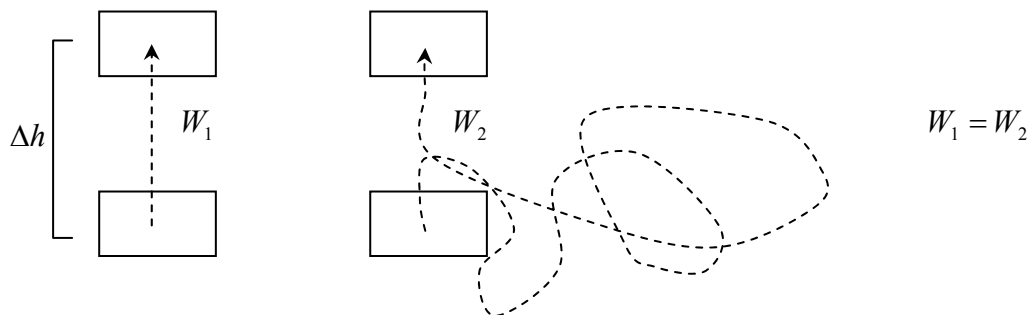
**Example 2:** How much work is required to lower a 30kg object from a height of 40m to a height of 15m?

$$\begin{aligned} W &= mg\Delta h \\ &= (30\text{kg})(9.8\text{N/kg})(15\text{m} - 40\text{m}) \\ &= -7350\text{N}\cdot\text{m} \\ &= -7350\text{J} \end{aligned}$$

The negative sign implies that energy is lost from the system. This result may seem surprising if one does not realize that the gravitational potential energy lost ( $-E_g$ ) is converted to another form, in this case kinetic energy ( $+E_k$ ) this is why dropped object acquire speed. (more on this later)

### More concepts on $E_g$

Based on the equation,  $W = \Delta E_g = mg\Delta h$ , work is only dependent on the change in height, not the path the object takes to get to that height.



### Bulk Kinetic Energy

Bulk kinetic energy is the energy of motion and is defined by the following formula

$$E_k = \frac{1}{2}mv^2$$

Where  $E_k$  is the kinetic energy in Joules ( $J$ ),  
 $m$  is the mass in  $kg$ , and  
 $v$  is the speed in  $m/s$

Therefore  $E_k$  is directly proportional to the mass of the object and directly proportional to the square of the speed of the object. E.g. a 30g projectile doesn't sound intimidating until it is launched at a speed of 400 m/s, such as the case with a gun. No one has ever been killed by a thrown bullet. Or consider tennis vs. hockey, no jocks required for tennis, double entendre notwithstanding.

**Work and Kinetic Energy**

The following derivation will show how work is related to kinetic energy. A few things to note, the following derivation is based on non-uniform motion since an unbalanced force will cause acceleration. Consider the

$$\begin{aligned}
 W &= F \cdot d \\
 &= \overbrace{mad}^{F = ma} \cdot \overbrace{d = \left(\frac{v_1 + v_2}{2}\right)\Delta t} \\
 &= m \left(\frac{\Delta v}{\Delta t}\right) \left(\frac{v_1 + v_2}{2}\right) \Delta t \\
 &= m \left(\frac{v_2 - v_1}{\Delta t}\right) \left(\frac{v_2 + v_1}{2}\right) \Delta t \\
 &= \frac{1}{2} m (v_2 - v_1)(v_2 + v_1) \text{ difference of squares} \\
 &= \frac{1}{2} m (v_2^2 - v_1^2) \\
 &= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \\
 &= E_{k_2} - E_{k_1} \\
 \therefore W &= \Delta E_k
 \end{aligned}$$

circumstance where an object of mass  $m$  is accelerated from a speed of  $v_1$  to a speed of  $v_2$ .

**Example:** How much  $E_k$  does a 50kg mass have if it is traveling at

- 0 m/s
- 2 m/s
- 4 m/s

a)

$$\begin{aligned}
 E_k &= \frac{1}{2} m v^2 \\
 &= \frac{1}{2} (50 \text{ kg})(0 \text{ m/s})^2 \\
 &= 0 \text{ kg} \cdot \text{m}^2 / \text{s}^2 \\
 &= 0 \text{ J}
 \end{aligned}$$

b)

$$\begin{aligned}
 E_k &= \frac{1}{2} m v^2 \\
 &= \frac{1}{2} (50 \text{ kg})(2 \text{ m/s})^2 \\
 &= 100 \text{ kg} \cdot \text{m}^2 / \text{s}^2 \\
 &= 100 \text{ J}
 \end{aligned}$$

c)

$$\begin{aligned}
 E_k &= \frac{1}{2} m v^2 \\
 &= \frac{1}{2} (50 \text{ kg})(4 \text{ m/s})^2 \\
 &= 400 \text{ kg} \cdot \text{m}^2 / \text{s}^2 \\
 &= 400 \text{ J}
 \end{aligned}$$

**Conservation of Energy**

In any closed system (meaning that energy does not leave or enter the system), the total amount of energy is conserved. In other words the sum of all the different types of energies in a system is a constant, implying that if one form of energy decreases, another form must increase.

**Example:** an object dropped from rest from the top of a building will lose its  $E_g$  but will gain its equivalent in  $E_k$ . Hence the taller the building the more  $E_g$  is lost resulting in a greater increase in  $E_k$  resulting in a larger impact velocity.

Consider the above situation with a building that is 100m tall.

$$E_{T_1} = E_{T_2}$$

$$E_{k_1} + E_{g_1} = E_{k_2} + E_{g_2} \quad \text{Note: } E_{k_1} = 0J \text{ because object is dropped from rest } (v_1 = 0m/s)$$

$$0 + mgh = \frac{1}{2}mv_2^2 + 0$$

$$E_{g_2} = 0J \text{ because object is on the ground } (h = 0m)$$

$$mgh = \frac{1}{2}mv_2^2$$

$$\frac{1}{2}mv_2^2 = mgh$$

$$\frac{1}{2}v_2^2 = gh$$

$$v_2^2 = 2gh$$

$$v_2 = \sqrt{2gh}$$

$$v_2 = \sqrt{2(9.8m/s^2)(100m)}$$

$$v_2 = \sqrt{1960m^2/s^2}$$

$$v_2 = 44.3m/s$$

$$\therefore \vec{v}_2 = 44.3m/s[D]$$

### Power and Work

Consider the following scenario. Two roofers are required to carry 30kg of shingles up to the second floor of a house using a ladder. The first roofer makes it to the second floor in 2 minutes where the other takes 5 minutes. According to the physics definition of work, both roofers performed equivalent amounts of **work** but clearly there is a difference in each roofer's performance. The first roofer completed the task in less than half the time it took for the second roofer. Clearly the first roofer is stronger thus more powerful.

$$P = \frac{W}{\Delta t} \quad \text{Where } W \text{ is work in Joules } (J) \text{ and}$$

$$\Delta t \text{ is the time interval, in seconds } (s), \text{ in which the work was done.}$$

$$P \text{ is the power in Watts } (W)$$

**Example:** How much power is required to lift a 10kg mass 2m if it takes

- 1.0s
- 0.5s

a)

$$P = \frac{W}{\Delta t}$$

$$= \frac{\Delta E_g}{\Delta t}$$

$$= \frac{mg\Delta h}{\Delta t}$$

$$= \frac{(10kg)(9.8N/m)(2m)}{(1.0s)}$$

$$= 196J/s$$

$$= 196W$$

b)

$$P = \frac{W}{\Delta t}$$

$$= \frac{\Delta E_g}{\Delta t}$$

$$= \frac{mg\Delta h}{\Delta t}$$

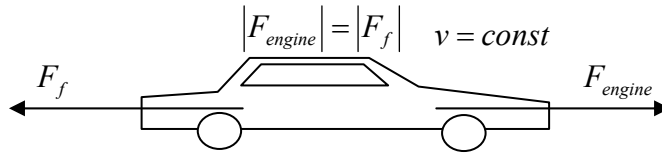
$$= \frac{(10kg)(9.8N/m)(2m)}{(0.5s)}$$

$$= 392J/s$$

$$= 392W$$

**Power and Speed**

In most everyday situations, most work is done to keep moving objects in motion. Hence most work done is against friction. Consider a car driving at a speed of 108km/h. To maintain this speed, the car must exert a force that counteracts the force of friction and wind resistance.



$$\begin{aligned}
 P &= \frac{W}{\Delta t} \\
 &= \frac{F \cdot \Delta d}{\Delta t} \\
 &= F \frac{\Delta d}{\Delta t} \\
 &= Fv
 \end{aligned}$$

$$\therefore P = Fv$$

A similar argument can be made if the speed is not constant but experiences linear acceleration

$$\begin{aligned}
 P &= \frac{W}{\Delta t} \\
 &= \frac{F \cdot \Delta d}{\Delta t} \\
 &= \frac{F (v_1 + v_2)}{2} \Delta t \\
 &= F \frac{(v_1 + v_2)}{2} \\
 &= Fv_{avg}
 \end{aligned}$$

**Example:** How much power is required to lift a 10kg object at a constant velocity of 2 m/s upwards.