Elastic collisions are collisions where the colliding objects do not:

1. Stick together
2. Permanently deform
3. Radiate energy in other forms

Consider 2 masses, m_1 and m_2. m_1 is moving at a constant initial velocity of v_1 and m_2 is at rest. Since we are working with collisions in one dimension, we do not need vector notation. We will simply use the standard that forward velocity is positive and reverse velocity is negative.

$E_{k_1} + E_{k_2} = E'_{k_1} + E'_{k_2} \quad (1)$ and $p_1 + p_2 = p'_1 + p'_2 \quad (2)$

Expressing (1) in terms of mass and velocity

$$E_{k_1} + E_{k_2} = E'_{k_1} + E'_{k_2}$$

$$\frac{m_1(v_1)^2}{2} + \frac{m_2(v_2)^2}{2} = \frac{m_1(v'_1)^2}{2} + \frac{m_2(v'_2)^2}{2}$$

$$m_1(v_1)^2 = m_1(v'_1)^2 + m_2(v'_2)^2$$

$$m_1(v_1)^2 - m_1(v'_1)^2 = m_2(v'_2)^2$$

$$\boxed{m_1\left((v_1)^2 - (v'_1)^2\right) = m_2(v'_2)^2} \quad (1a)$$

Expressing (2) in terms of mass and velocity

$$p_1 + p_2 = p'_1 + p'_2$$

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$

$$m_1 v_1 - m_1 v'_1 = m_2 v'_2$$

$$\boxed{m_1(v_1 - v'_1) = m_2 v'_2} \quad (2a)$$

$$(1a) \div (2a)$$

$$\frac{m_1\left((v_1)^2 - (v'_1)^2\right)}{m_1(v_1 - v'_1)} = \frac{m_2(v'_2)^2}{m_2 v'_2}$$

$$\frac{(v_1)^2 - (v'_1)^2}{(v_1 - v'_1)} = v'_2$$

$$\frac{(v_1 - v'_1)(v_1 + v'_1)}{(v_1 - v'_1)} = v'_2$$

$$v_1 + v'_1 = v'_2$$

$$\boxed{v_1 + v'_1 = v'_2} \quad (3)$$

Sub (3) into (2)

$$p_1 + p_2 = p_1' + p_2'$$

$$m_1v_1 + \cancelto{0}{m_2v_2} = m_1v_1' + m_2v_2'$$

$$m_1v_1 = m_1v_1' + m_2(v_1 + v_1')$$

$$m_1v_1 = m_1v_1' + m_2v_1 + m_2v_1'$$

$$m_1v_1 - m_2v_1 = m_1v_1' + m_2v_1'$$

$$v_1(m_1 - m_2) = v_1'(m_1 + m_2)$$

$$v_1\frac{(m_1 - m_2)}{(m_1 + m_2)} = v_1'$$

$$\boxed{\therefore v_1' = \frac{(m_1 - m_2)}{(m_1 + m_2)}v_1}$$

Sub (3) into (2) using this form $v_1' = v_2' - v_1$

$$p_1 + p_2 = p_1' + p_2'$$

$$m_1v_1 + \cancelto{0}{m_2v_2} = m_1v_1' + m_2v_2'$$

$$m_1v_1 = m_1(v_2' - v_1) + m_2v_2'$$

$$m_1v_1 = m_1v_2' - m_1v_1 + m_2v_2'$$

$$m_1v_1 + m_1v_1 = m_1v_2' + m_2v_2'$$

$$(m_1 + m_1)v_1 = (m_1 + m_2)v_2'$$

$$(2m_1)v_1 = (m_1 + m_2)v_2'$$

$$\frac{(2m_1)}{(m_1 + m_2)}v_1 = v_2'$$

$$\boxed{\therefore v_2' = \frac{(2m_1)}{(m_1 + m_2)}v_1}$$

Summary of Derived Equations

1. $\boxed{v_1 + v_1' = v_2'}$

2. $\boxed{v_1' = \frac{(m_1 - m_2)}{(m_1 + m_2)}v_1}$

3. $\boxed{v_2' = \frac{(2m_1)}{(m_1 + m_2)}v_1}$

Note:

These equations only work with elastic collisions in one direction assuming that m_2 is initially at rest.

In any other circumstance, these equations cannot be used. If m_2 is not at rest, you may still use these equation but you have to use relative motion to set the velocity of m_2 to zero and then convert the velocities back again.

Example Questions:

1. A cart of mass 2.0kg is moving to the right along a smooth, horizontal track at 3.0m/s. A spring, with a force constant of 1200N/m and a nominal length of 25cm, is attached to the front of the cart. The 2.0kg cart then collides into a 4.0kg cart that is initially at rest. Find
   a) The velocities of both carts after the collision
   b) The velocities of the carts at minimum separation
   c) The amount of E_k lost at minimum separation, explain where the energy went
   d) The minimum separation of the carts.

2. A massless spring is compressed between two blocks of mass m and 5m respectively. Find the fraction of the spring's energy shared between the two blocks.