Momentum is an extension of Newton's 2nd law.

When analyzing an object's motion there are two factors to consider when attempting to bring it to rest.

1. The object's mass
2. The object's velocity

The greater an object's mass, the more force is required to bring it to rest. The same holds true for the velocity.

By definition momentum is the product of an object's mass with its velocity.

$$\vec{p} = m\vec{v}$$

Where $\vec{p}$ is the momentum in $kg \cdot m/s$.
m is the mass in kg
and $\vec{v}$ is the velocity in m/s

Note: The product of a scalar and a vector is always a vector. The direction of the product is always in the direction of the vector. Therefore momentum is always in the direction of the velocity.

Examples:

1. A freight train traveling at 60km/h has much more momentum then a compact car traveling at 60km/h
2. Conversely, a 30g bullet traveling at 400m/s has more momentum than a 0.5kg stone thrown at 72km/h.

Newton described momentum as the "quality of motion". An object's momentum, at any given velocity, is related to the object's mass and hence, the object **inertia**. (recalling that inertia is an object's inherent resistance to change in motion)

EX1: What is the momentum of

a) a 30g bullet traveling at 400m/s
b) a 0.5kg stone thrown at 72km/h.

Given	RTF	Formula
$m_b = 30g = 0.03kg$ $m_s = 0.5kg$ $\vec{v}_b = 400\,m/s$ $\vec{v}_s = 72\,km/h \rightarrow 20\,m/s$	a) $\vec{p}_b$ b) $\vec{p}_s$	$\vec{p} = m\vec{v}$

Solution

a)
$$\begin{aligned}\vec{p} &= m\vec{v}\\ &= (0.03)(400)\\ &= 12\,kg \cdot m/s\\ \vec{p}_b &= 12\,kg \cdot m/s\,[?]\end{aligned}$$

b)
$$\begin{aligned}\vec{p} &= m\vec{v}\\ &= (0.5)(20)\\ &= 10\,kg \cdot m/s\\ \vec{p}_s &= 10\,kg \cdot m/s\,[?]\end{aligned}$$

Impulse and Momentum

Impulse is defined as the change of momentum of an object or simply

$$\Delta\vec{p}$$

by definition

$$\Delta\vec{p} = m\vec{v}_2 - m\vec{v}_1$$

EX: Find the impulse acting on a 1000kg car that slows from 108km/h [E] to 54 km/h [E]

Impulse and Newton's 2nd Law

Newton originally expressed his second law as the change in momentum over time.

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

The **impulse** can therefore be expressed in terms of force and time

$$\Delta\vec{p} = \vec{F}\Delta t$$

Where $\vec{F}$ is the force in N and Δt is the time interval, in seconds, for which the force acts.

Note: this definition assumes a **constant force.** Therefore the change in momentum is directly proportional to the applied force and the duration.

Relating the two forms of Newton's Second Law

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} \quad \text{and} \quad \vec{F} = ma$$

Proof

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

As defined earlier $\Delta\vec{p} = m\vec{v}_2 - m\vec{v}_1$

$$\vec{F} = \frac{m\vec{v}_2 - m\vec{v}_1}{\Delta t}$$

Pulling out a common factor of m

$$\vec{F} = \frac{m(\vec{v}_2 - \vec{v}_1)}{\Delta t}$$

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

$$\vec{F} = \frac{m(\Delta\vec{v})}{\Delta t}$$

Rearranging

$$\vec{F} = m\frac{(\Delta\vec{v})}{(\Delta t)}$$

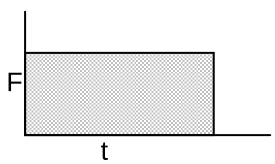
$$\vec{F} = m\vec{a}$$

Finding the Impulse for Non-Constant Forces

One can **find** the **impulse** acting on an object by **graphical analysis**. The idea is similar to how one can find the **displacement** from a **v vs. t** graph.

Impulse is defined as the **area beneath a F vs. t graph.**

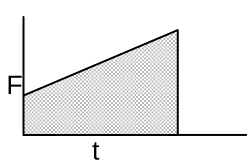
<u>Force is Constant</u>



$$A = l \times w$$

$$\Delta\vec{p} = \vec{F}\Delta t$$

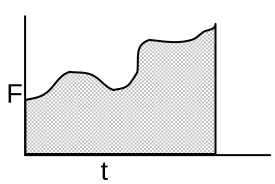
<u>Force Changes Uniformly</u>



$$A = \frac{(a+b)}{2}h$$

$$\Delta\vec{p} = \frac{(F_1 + F_2)}{2}\Delta t$$

<u>Force Changes Non-Uniformly</u>



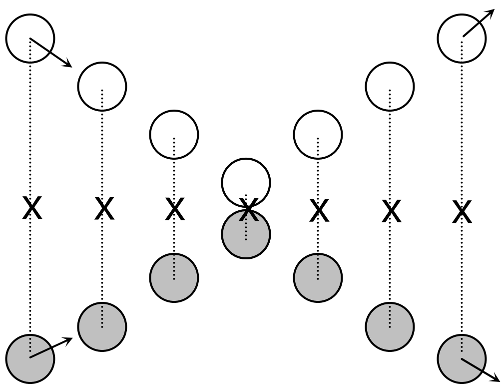
$$\Delta p = \int_{t_1}^{t_2} F(t)\,dt$$

University level mathematics required, therefore outside of the scope of this course… relax!

Conservation of Momentum

Concept: The total momentum within a closed system is constant. This implies that the total momentum before and after a collision is constant. Meaning that when two or more objects collide, resulting in a change in velocity for each object, the total momentum before and after the collision will still be constant.

$$\vec{p}_{total} = \vec{p}'_{total}$$



The diagram to the left shows two objects colliding.

The **X** represents the motion of the **center of mass** of the system.

Notice that the center of mass continues to move with constant motion, despite the collision between the objects within the system

$$\therefore\ \vec{p}_{total} = \vec{p}'_{total} = \vec{p}_{cm}$$

Proving Newton's 3rd Law Using Momentum

Consider two masses m_1 and m_2 initially traveling at v_1 and v_2 respectively

$$\begin{aligned}
\vec{p}_{total} &= \vec{p}'_{total} \\
m_1\vec{v}_1 + m_2\vec{v}_2 &= m_1\vec{v}'_1 + m_2\vec{v}'_2 \\
m_1\vec{v}_1 - m_1\vec{v}'_1 &= m_2\vec{v}'_2 - m_2\vec{v}_2 \\
-m_1\vec{v}'_1 + m_1\vec{v}_1 &= m_2\vec{v}'_2 - m_2\vec{v}_2 \\
-\left(m_1\vec{v}'_1 - m_1\vec{v}_1\right) &= m_2\vec{v}'_2 - m_2\vec{v}_2 \\
-m_1\left(\vec{v}'_1 - \vec{v}_1\right) &= m_2\left(\vec{v}'_2 - \vec{v}_2\right) \\
-m_1\left(\Delta\vec{v}_1\right) &= m_2\left(\Delta\vec{v}_2\right)
\end{aligned}$$

Rearranging equations so that the left side contains all the m_1 terms and the right side contains all the m_2 terms.

Rearranging equations so both sides of the equations will be expressed having the v' term first (final velocity) and the v term second (initial velocity).

Since the collision occurred over a time interval of Δt , both sides of the equation can be divided by Δt

$$-\frac{m_1\left(\Delta\vec{v}_1\right)}{\Delta t} = \frac{m_2\left(\Delta\vec{v}_2\right)}{\Delta t} \qquad \vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

$$-m_1\vec{a}_1 = m_2\vec{a}_2 \qquad \text{Newton's 2nd Law}$$

$$-F_1 = F_2 \qquad \text{Newton's 3rd Law}$$

Therefore the force acting on m_1 is equal and opposite to the force acting on m_2

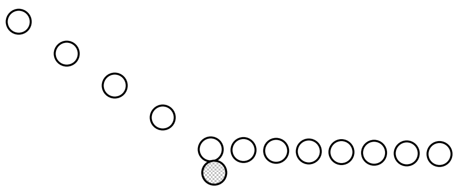
Examples

1. A $150\tilde{0}\,\text{kg}$ vehicle is driving east at 126km/h.
   a. Find the momentum of the car.
   b. Find the change in momentum of the car if is slows to 90km/h.
   c. Find the impulse of the car if it slows down to 90km/h.
   d. How much force was required to slow the vehicle down if it took 5s for the car to slow down to 90km/h.

2. A 1.0kg toy car runs head on into a spring that is attached to a wall. If the spring compresses 5cm in 2 seconds, what would be the k for the spring if the car was moving at 1.0m/s and temporarily stopped when the spring reached its maximum compression?

3. A 100g ball, moving at a constant velocity of 0.2 m/s, runs into a stationary 400g ball. If the first ball bounces straight back at a rate of 0.12m/s, find the final velocity of the second ball.

4. Predict the path of the second ball after the collision using your knowledge how the center of mass behaves after a collision.



5. A white1.0kg ball, traveling at 2.0m/s [R20^{o}D], collides with a 2.0kg ball traveling at 2.5m/s [R10^{o}U] as show below. If after the collision, the gray ball is deflected down so that it moves at 2.0m/s [R55^{o}D], find the velocity of the white ball.