Kinetic energy is the energy of motion. We say motion has energy because moving objects have the ability to do work. (e.g. a hammer hitting a nail, a baseball bat hitting a ball)

Kinetic energy is measured in Joules (J).

Work:

$W = \vec{F} \cdot \Delta\vec{d}$ → This is a force through a displacement. Note: work is the dot product between the force in Newtons (N) and the displacement in meters (m). The result is a scalar.

Or

$W = F \cdot \Delta d \cos\theta$ → Where F is the force in Newtons (N), Δd is the displacement in meters (m) and θ is the angle between the Force and the displacement.

Note: $W = 0J$ if $\vec{F} \perp \Delta\vec{d}$

Deriving Kinetic Energy:

Work is defined as the change in kinetic energy

$W = \vec{F} \cdot \Delta\vec{d}$ (1) → From momentum we know $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ (2)

sub (2) into (1)

$$W = \vec{F} \cdot \Delta\vec{d}$$

$$W = \frac{\Delta\vec{p}}{\Delta t} \cdot \Delta\vec{d}$$

$$W = \frac{\Delta\vec{p}}{\Delta t} \cdot (\frac{\vec{v}_1 + \vec{v}_2}{2})\Delta t$$

← From kinematics remember $\Delta\vec{d} = (\frac{\vec{v}_1 + \vec{v}_2}{2})\Delta t$

$$W = \frac{(m\Delta\vec{v})}{\Delta t} \cdot (\frac{\vec{v}_1 + \vec{v}_2}{2})\Delta t$$

← $\Delta\vec{p} = m\Delta\vec{v}$

$$W = \frac{m(\vec{v}_2 - \vec{v}_1)}{\Delta t} \cdot (\frac{\vec{v}_1 + \vec{v}_2}{2})\Delta t$$

$$W = \frac{m(\vec{v}_2 - \vec{v}_1)(\vec{v}_1 + \vec{v}_2)\Delta t}{2\Delta t}$$

← Notice we drop the vector notation. This is because when multiplying through the brackets, we are working with the dot products, which render a scalar result.

$$W = \frac{m({v_2}^2 - {v_1}^2)}{2}$$

$$W = \frac{m{v_2}^2}{2} - \frac{m{v_1}^2}{2}$$

$$\therefore W = E_{k_2} - E_{k_1}$$

$$\therefore W = \Delta E_k$$

In general $$E_k = \frac{1}{2}mv^2$$

EX: You push a 10kg block for 15m with a constant force of 20N.

a) How much work did you do?
b) If the block started from rest, how fast is it moving at the 15m mark?

We can relate E_k to momentum using algebraic manipulation

$E_k = \frac{1}{2}mv^2$ (1) and $p = mv \rightarrow v = \frac{p}{m}$ (2)

Sub (2) into (1)

$$E_k = \frac{1}{2}m\left(\frac{p}{m}\right)^2$$

$$E_k = \frac{1}{2}\frac{mp^2}{m^2}$$

$$\boxed{E_k = \frac{p^2}{2m}} \quad \text{or} \quad \boxed{p = \sqrt{2mE_k}}$$

<u>Conservation of Energy and Conservation of Momentum</u>

Conservation of energy implies that energy cannot be created or destroyed. It only transformed into another forms.

Conservation of momentum implies that the total momentum within a system is constant.

<u>With respect to collisions</u>

- Momentum is **<u>ALWAYS</u>** conserved
- Kinetic energy is **<u>NOT</u>** conserved **<u>unless</u>** the collision is **ELASTIC**

With **<u>elastic collisions</u>** the two colliding objects **<u>DO NOT</u>**

- Stick together
- Permanently deform
- Radiate energy in other forms

In reality most collisions are **not** elastic, but some collision are approximately elastic, meaning that one can use the assumption of conservation of energy for some types of situations that approximate an elastic collision.