Kinematics

Kinematics is an area of physics where the preciseness of language is very important. It is absolutely crucial to read all questions carefully, because many times what is asked and what is one's interpretation of what is being asked, can be the difference between getting the answer and getting frustrated.

Velocity as Vector – Speed as Scalar

Average Speed- The **total distance** divided by the **total time** to travel that distance.

$$v_{av} = \frac{d_{total}}{t_{total}}$$

Instantaneous Speed - The **infinitesimally small change in distance** divided by the **infinitesimally small amount of time** required to travel that distance.

$$v_{inst} = \lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t}$$

Average Velocity - The **total displacement** divided by the **total time** to travel that displacement expressed as a **VECTOR!**

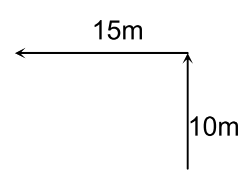
$$v_{av} = \frac{\vec{d}_{total}}{t_{total}}$$

Instantaneous Velocity - the **infinitesimally small change in displacement** divided by the **infinitesimally small amount of time** required to travel that displacement expressed as a **V E C T O R!**

$$\vec{v}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t}$$

Note: the numerical values for the instantaneous speed and velocity are the same.

Example: Michael walks north at a constant rate. In 10 seconds he walks 10m. He then west and walks an additional 15m at a constant rate for 10 seconds.



a) What was his average speed over the first 10s?

$$v_{av} = \frac{\Delta d_t}{\Delta t}$$

$$v_{av} = \frac{10m - 0m}{10s - 0s}$$

$$v_{av} = 1 m/s$$

b) What was his instantaneous speed at 7s?

Since he is traveling at a constant speed, instantaneous speed along that section is equal to the average speed, therefore

$$v_{inst} = 1 m/s$$

c) What was his average velocity over the first 10s?

$$\vec{v}_{av} = \frac{\Delta \vec{d}_t}{\Delta t}$$

$$\vec{v}_{av} = \frac{10m[N] - 0m[N]}{10s - 0s}$$

$$\vec{v}_{av} = 1 m/s[N]$$

d) What was his instantaneous velocity at 7s?

Since he is traveling at a constant velocity, instantaneous velocity along that section is equal to the average velocity, therefore

$$\vec{v}_{inst} = 1 m/s[N]$$

e) What was his average speed over the 20s?

$$v_{av} = \frac{\Delta d_t}{\Delta t}$$

$$v_{av} = \frac{25m - 0m}{20s - 0s}$$

$$v_{av} = 1.25 m/s$$

f) What was his average speed over the last 10s?

$$v_{av} = \frac{\Delta d_t}{\Delta t}$$

$$v_{av} = \frac{15m - 0m}{20s - 10s}$$

$$v_{av} = 1.5\, m/s$$

g) What was his instantaneous speed at 12s?

Since he is traveling at a constant speed, instantaneous speed along that section is equal to the average speed, therefore

$$v_{inst} = 1.5\, m/s$$

h) What was his average velocity over the last 10s?

$$\vec{v}_{av} = \frac{\Delta \vec{d}_t}{\Delta t}$$

$$\vec{v}_{av} = \frac{15m[W] - 0m[W]}{20s - 10s}$$

$$\vec{v}_{av} = 1.5\, m/s[W]$$

i) What was his instantaneous velocity at 12s?

Since he is traveling at a constant velocity, instantaneous velocity along that section is equal to the average velocity, therefore

$$\vec{v}_{inst} = 1.5\, m/s\,[W]$$

j) What was his **average velocity** over the 20s?

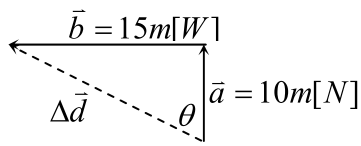
$\vec{v}_{av} = \frac{\Delta \vec{d}_t}{\Delta t}$ This question **must** be solved using vector addition

let $\vec{a} = 10m[N]$ represents the displacement in the first 10s and $\vec{b} = 15m[W]$ represent the displacement in the last 10s.

$$\Delta \vec{d}_t = \vec{b} + \vec{a}$$

$\because \vec{a} = 10m[N]$ represents the displacement in the first 10s and $\vec{b} = 15m[W]$ represent the displacement in the last 10s

$$\Delta \vec{d}_t = \vec{b} + \vec{a}$$



Find $|\Delta \vec{d}|$ (absolute values bars means magnitude)

$$\Delta \vec{d} = \sqrt{|\vec{a}|^2 + |\vec{b}|^2}$$

$$= \sqrt{(10m)^2 + (15m)^2}$$

$$= 18m$$

Find θ

$$\tan\theta = \frac{|\vec{b}|}{|\vec{a}|}$$

$$\tan\theta = \frac{15}{10}$$

$$\theta = \tan^{-1}(1.5)$$

$$\theta = 56^\circ$$

Find $|\vec{v}_{av}|$

$$|\vec{v}_{av}| = \frac{|\Delta \vec{d}|}{\Delta t}$$

$$= \frac{18m}{20s}$$

$$= 0.9\, m/s$$

$$\therefore \vec{v}_{av} = 0.9\, m/s[N56^\circ W]$$

Acceleration as a Vector

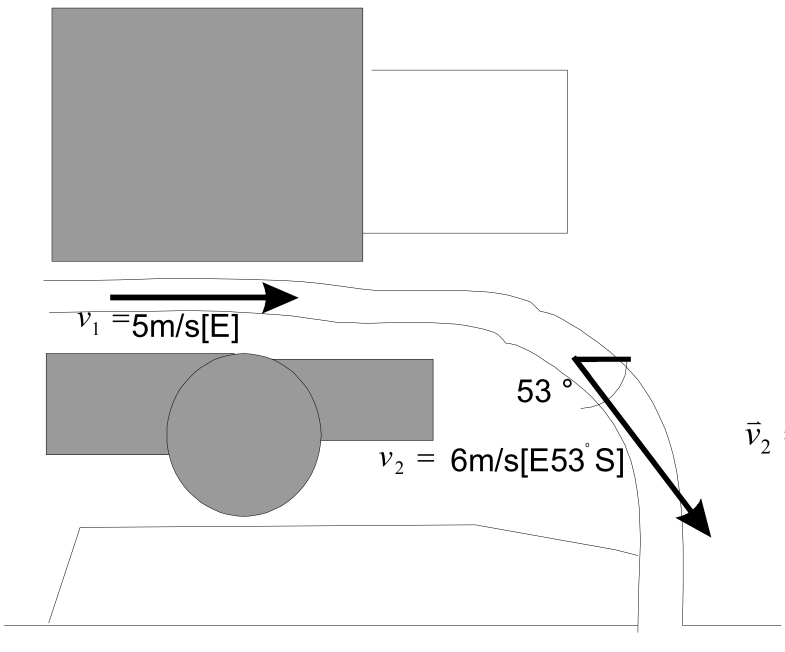
Since both **velocity** and **displacement** are vectors, our calculation of **acceleration** must also be a vector.

These are the definitions for **average** and **instantaneous acceleration**. In order to determine $\Delta\vec{v}$? **Vector addition** and / or **subtraction** must be used.

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

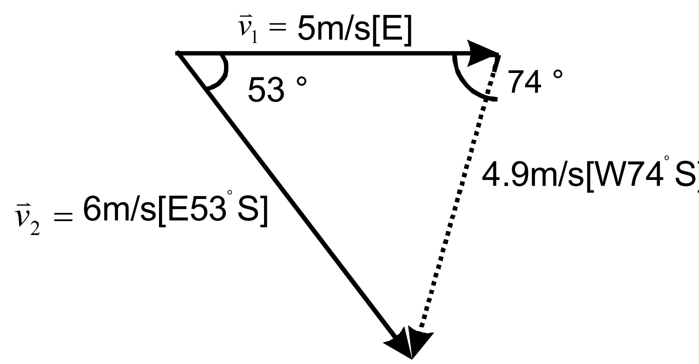
Example: Norm is trying to impress everybody by showing the staff and students of St. Mary's, the true performance of his Dodge Omni. To do this, He decides to "race" his car on the one way street behind the school. Here is an overhead shot of his performance.



To figure out Norm's **acceleration** we have to figure out his change in velocity.

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$

Since the velocities are expressed as vectors it's useful to create the vector diagrams.



Using **cosine law** $\rightarrow \left|\Delta\vec{v}\right| = \sqrt{5^2 + 6^2 - 2(5)(6)\cos(53^\circ)} = 4.9m/s$.

The angle θ can be determined using **sine law** $\rightarrow \frac{6}{\sin\theta} = \frac{4.9}{\sin(53^\circ)} \rightarrow \theta = 74^\circ$

$\vec{\Delta}v = 4.9m/s[W74^\circ S]$. Say it took him 20s to achieve that change in velocity. His **acceleration** would be

$$a = \frac{4.9m/s}{20s}$$
$$= 0.245m/s^2$$
$$\vec{a} = 0.245m/s^2[W74^\circ S]$$

EXERCISE: From the previous example, determine the Michael's acceleration after 20s.