Projectile motion is the branch of kinematics that predicts the behaviour of objects launched under the influence of a gravitational field. Project motion can be broken up into two **components**, the **vertical** (**y**) and the **horizontal** (**x**)

Rules for Projectile motion

1. For simplicity sake, all air resistance is ignored
2. Motion in **vertical** component is affected by gravity, and hence, is governed by the rules for accelerated motion.
3. Motion in the **horizontal** component experiences no acceleration, and hence, is governed by the rules for linear motion. The **horizontal displacement** is called the **range**.
4. The rise time for a projectile is equal to the fall time.
5. The flight time, for both horizontal and vertical components, are the same.

Proving Rule 4

A ball is shot straight up with an initial velocity of $49m/s[U]$. Find

a) How long it takes to reach the top
b) How high it rises
c) How long it takes to fall
d) Impact velocity

Given	RTF	Formula
$v_1 = 49m/s[U]$ $a = 9.8m/s^2[D]$	a) How long it takes to reach the top b) How high it rises c) How long it takes to fall d) Impact velocity	$a = \frac{\Delta v}{\Delta t}$ $d = v_2 t - \frac{1}{2}at^2$ $d = v_1 t + \frac{1}{2}at^2$

Solution

a) **Rise time:** When the projectile reaches its maximum height, it will temporarily come to rest. Therefore, $v_2 = 0m/s$

$$a = \frac{\Delta v}{\Delta t}$$
$$\Delta t = \frac{\Delta v}{a}$$
$$\Delta t = \frac{v_2 - v_1}{a}$$
$$\Delta t = \frac{0-49}{-9.8}$$
$$\Delta t = 5s$$

∴ Rise time is 5s

b) **Max height:**

$$d = v_2 t - \frac{1}{2}at^2$$
$$d = (0)(5) - \frac{1}{2}(-9.8)(5)^2$$
$$d = 122.5m$$

∴ maximum height is 122.5m

c) **Fall Time:** The projectile falls 122.5m to the ground with an initial velocity of 0m/s

$$d = v_1 t + \frac{1}{2}at^2$$
$$-122.5 = (0)t + \frac{1}{2}(-9.8)t^2$$
$$-122.5 = (-4.9)t^2$$
$$t^2 = \frac{-122.5}{-4.9}$$
$$t = \sqrt{25}$$
$$t = \pm 5s$$

∴ Fall time is 5s

d) **Impact Velocity:** Calculate impact velocity by using the fall only. Therefore $v_1 = 0m/s$

$$a = \frac{\Delta v}{\Delta t}$$
$$\Delta v = a\Delta t$$
$$v_2 - v_1 = a\Delta t$$
$$v_2 - (0) = (-9.8)(5)$$
$$v_2 = -49m/s$$

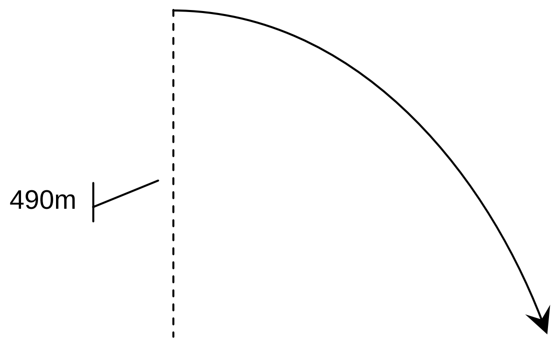
∴ Impact velocity is $49m/s[D]$

Example: An object is launched horizontally from a height of 490m at an initial velocity of 10m/s [R]. (Assuming no air resistance). Find:

a) the flight time
b) horizontal displacement
c) total displacement

a) Recall:

- treat the **horizontal** and **vertical** components **separately**
- **vertical component** is **non linear motion**
- **horizontal component** is **linear motion**
- both the horizontal and vertical components occur during the same time period so **t** is the same for both



Given	**RTF**	**Formulae**
$\vec{v}_{y_1} = 0m/s$ $\vec{d}_y = -490m$ $\vec{v}_x = 10m/s[R]$ $\vec{a}_y = -9.8m/s^2$	a) Flight time b) Horz displacement c) Total displacement	$\vec{d} = \vec{v}_1 t + \frac{1}{2}\vec{a}t^2$ $\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$

<u>Find time from vertical component</u>

$$\vec{d} = \vec{v}_1 t + \tfrac{1}{2}\vec{a}t^2$$
$$-490 = (0)t + \tfrac{1}{2}(-9.8)(t)^2$$
$$-490 = -4.9t^2$$
$$100 = t^2$$
$$t = 10s$$

∴ flight time is 10s

b) **<u>Find horizontal displacement</u>**

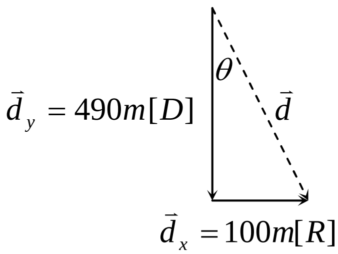
Recall: flight time is constant for both vertical and horizontal components

$$\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$$
$$\Delta\vec{d} = \vec{v}\Delta t$$
$$\Delta\vec{d} = (10)(10)$$
$$\Delta\vec{d} = 100m$$

∴ horizontal displacement is 100m

c) **Find total displacement**

$$\vec{d} = \vec{d}_y + \vec{d}_x$$



Find $|\vec{d}|$

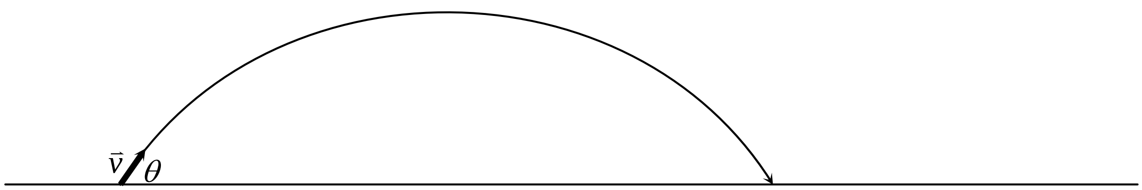
$$|\vec{d}| = \sqrt{|d_y|^2 + |d_x|^2}$$
$$= \sqrt{(490)^2 + (100)^2}$$
$$= 500m$$

Find θ

$$\tan\theta = \frac{d_x}{d_y}$$
$$= \frac{100}{490}$$
$$\theta = \tan^{-1}(\frac{100}{490})$$
$$\theta = 11.5^\circ$$
$$\theta = 12^\circ$$

$$\therefore \vec{d} = 500m[D12^\circ R]$$

The Projectile Motion Equations



Facts about projectile motion

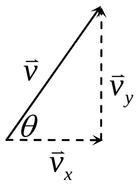
1) What goes up must come down
2) If there is no air resistance, the trajectory is **always** a parabola
3) Rise time = fall time (assuming level ground)
4) If there is no air resistance, horizontal velocity is constant

Derivation

These derived equations will only work in situations where

- there is no consideration to air resistance
- there is no acceleration in the x-component
- the projectile is launched from a smooth, flat surface

Consider both vertical and horizontal components of a launch



Vertical Component

Given:

$\vec{a}_y = g[D]\ (9.8m./s^2)$

$\vec{d}_y = 0m$ (Why? What goes up, must come down)

$\vec{v}_y = v\sin(\theta)[U]$

Let [U] be positive and [D] be negative

$$\vec{d}_y = \vec{v}_{1_y}t + \frac{1}{2}\vec{a}_y t^2$$

$$0 = v\sin(\theta)t + \frac{1}{2}(-g)t^2$$

$$0 = v\sin(\theta)t - \frac{1}{2}(g)t^2$$

$$\frac{1}{2}(g)t^2 = v\sin(\theta)t\ \ (t's\ cancel)$$

$$\frac{1}{2}(g)t = v\sin(\theta)$$

$$(g)t = 2v\sin(\theta)$$

$$t = \frac{2v\sin(\theta)}{g}$$ ①

Horizontal Component

Given:

$\vec{v}_x = v\cos(\theta)$

Let [R] be positive and [L] be negative

$$\vec{d}_x = \vec{v}_x t$$

$$d_x = v\cos(\theta)t$$

concept: The time for the vertical component is the same as the time for the horizontal component because both the start points and end points are the same for both components. Hence we can use t from the **vertical component** to solve for $\vec{d}_x$

$$d_x = v\cos(\theta)t$$

$$d_x = v\cos(\theta)\left(\frac{2v\sin(\theta)}{g}\right)$$ ①

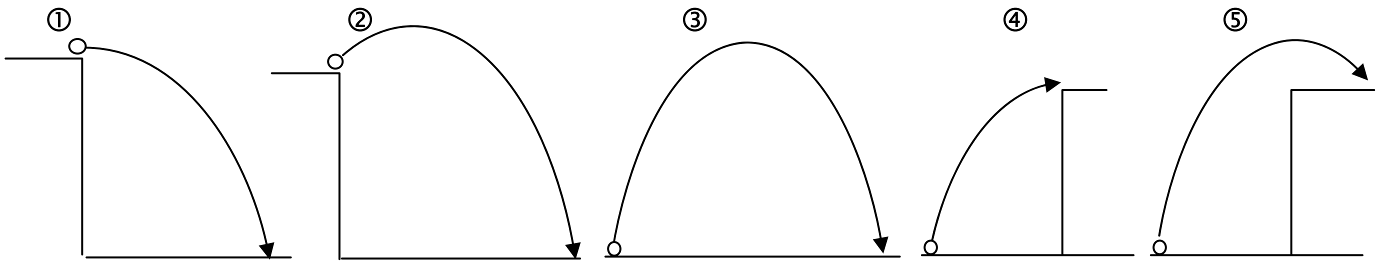
$$d_x = \frac{v^2 2\sin(\theta)\cos(\theta)}{g}$$

Here we must use the double angle theorem to solve the rest of the question

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$d_x = \frac{v^2\sin(2\theta)}{g}$$ ②

Typical Projectile Motion Questions



Type ①:

a) A projectile was launched horizontally at a velocity of 10m/s from the top of a cliff that is 50m high. Find the flight time and the range.

b) A projectile was launched horizontally at a velocity of 50 m/s from a cliff. If it takes 15s to land, find the range and the height of the cliff.

Type ②:

a) A projectile was launched with a velocity of $10m/s[R30^0U]$ from the top of a cliff that is 50m high. Find the flight time and the range.

b) A projectile was launched with a velocity of $20m/s[R30^0U]$ from the top of a cliff. Find the flight time and the height of the cliff if the range is 86.6m.

Type ③:

a) A projectile was launched with a velocity of $50m/s[R30^0U]$ from flat ground. Find the flight time, maximum height and the range.

b) A projectile launched at an unknown velocity, lands 35.3m away in 3.53s. Find the horizontal velocity, the max height. The initial vertical velocity. The launch velocity

Type ④:

a) What initial velocity is required for the projectile to reach the top of a 10m cliff if projectile is launch 20m from the base of the cliff?

Type ⑤:

a) A projectile is launched with an initial velocity of $50m/s[R60^0U]$ 60m from the bottom of a 20m cliff. How far along the plateau of the cliff does the projectile land?