In **grade 11**, you mostly dealt with kinematics in **one dimension**. In **grade 12**, however, the focus is on kinematics in **two dimensions**.

Review of Kinematics in One Dimension

Average Velocity – Not your *"average"* calculation!

Very rarely in the real world is velocity constant. It is thus, very convenient to talk about **average velocity**. By definition:

$$\textbf{The average velocity} = \frac{\textbf{The } \underline{\textbf{total}} \textbf{ displacement}}{\textbf{The } \underline{\textbf{total}} \textbf{ time}}$$

or

$$\vec{v}_{av} = \frac{\vec{d}_{total}}{t_{total}}$$

Example:
Aaron was driving on the 401 at 110km/h [E] for ½ hr. He then encountered a traffic jam, which reduced his speed to 50km/h for 1hr. What was his average speed?

$$v_{av} = \frac{110+50}{2} = \frac{160}{2} = 80 km/h[E]$$

BAD! BAD! BAD!

One can not simply take the average of the two velocities. It is the same flawed logic that many students use in determining their final grades. For example, averaging the mark on a major test with a mark on a quiz will not get you a valid result.

Consider the following: A student earns a mark of 10/10 on a quiz but only 50/100 on a major test on the friction unit in his grade 11 course. The flawed logic goes follows:

"But sir… I got 100% on the quiz and 50% on the test; that should be an average of 75%" he/she says in that incredulous tone that teenagers are so apt to have when incensed.

The teacher responds patiently "No actually! That is not how averaging works. You see, the test is worth much more, therefore is weighted more heavily. To correctly determine your average, you must first add together all your earned marks on both evaluations (10+50), and the totals on each evaluation (100+10). Then using these results, you find the average as follows … 60/110=0.5454 which is approximately 55%. Get it?"

Student responds, "Friction sucks!"

Annoyed, teacher responds smugly, with slight contempt – shamelessly indulging in self satisfaction for the brilliant and poignantly sarcastic witticism he is about unleash on his critic, "Actually its more of a dragging sensation"

The room falls dead. In silence, a perfectly executed zinger sinks quicker than a feather vacuum jar.

Back to physics….

The proper solution

$$\text{total time for trip} \quad \Delta t = 1h + 0.5h = 1.5h$$

$$\text{displacement at 110km/h} \quad d = vt$$

$$d = 110km/h \times 0.5h$$

$$= 55km[E]$$

$$\text{displacement at 50km/h} \quad d = 50km/h \times 1h$$

$$= 50km[E]$$

$$\text{average velocity} \quad v_{av} = \frac{50km + 55km}{1.5h}$$

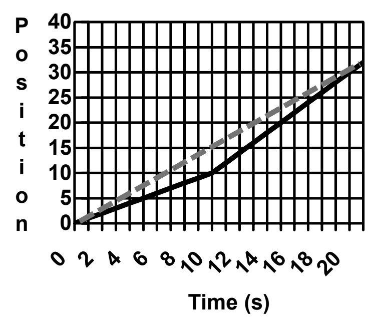
$$= \frac{105km}{1.5h}$$

$$= 70km/h[E]$$

Average Velocity and Position – Time Graphs

One can also find the average velocity from a **position-time graph**

To find the average velocity between 0 and 20s, take the slope of the dotted line



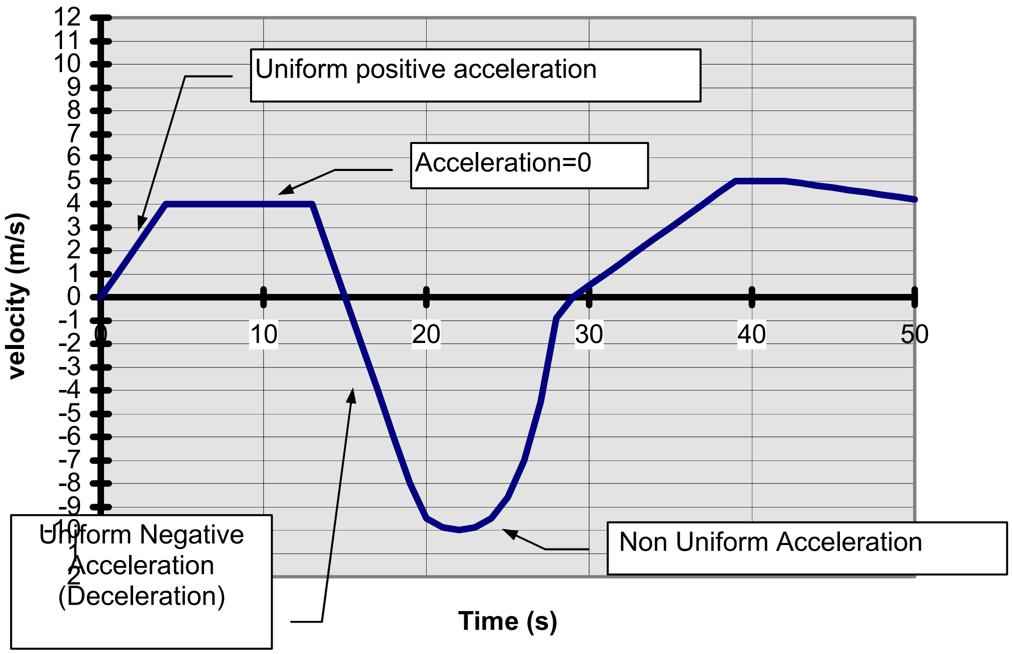
$$slope = v_{av} = \frac{\Delta d}{\Delta t} = \frac{30-0}{20-0} = 1.5m/s$$

The instantaneous velocity would be the slope at any given point on the graph.

Velocity-Time Graphs

Motion can also be represented by a velocity-time graph. This graph is exactly of the same structure as a position-time graph but it makes a plot of an object's **velocity** over a period of **time**. The shape of the curve indicates properties of the object's acceleration at any given time. Things to know about velocity-time graphs.

1) **Straight lines indicate uniform acceleration**
2) **Straight lines with positive slope indicate positive acceleration (accelerating forward)**
3) **Straight lines with negative slope indicate negative acceleration (deceleration)**
4) **The slope along any point of the curve on a velocity-time graph gives the acceleration of the object at that point**
5) **Curved lines on a velocity-time graph indicate non uniform acceleration**
6) **When the slope is zero the acceleration is zero (velocity is constant)**

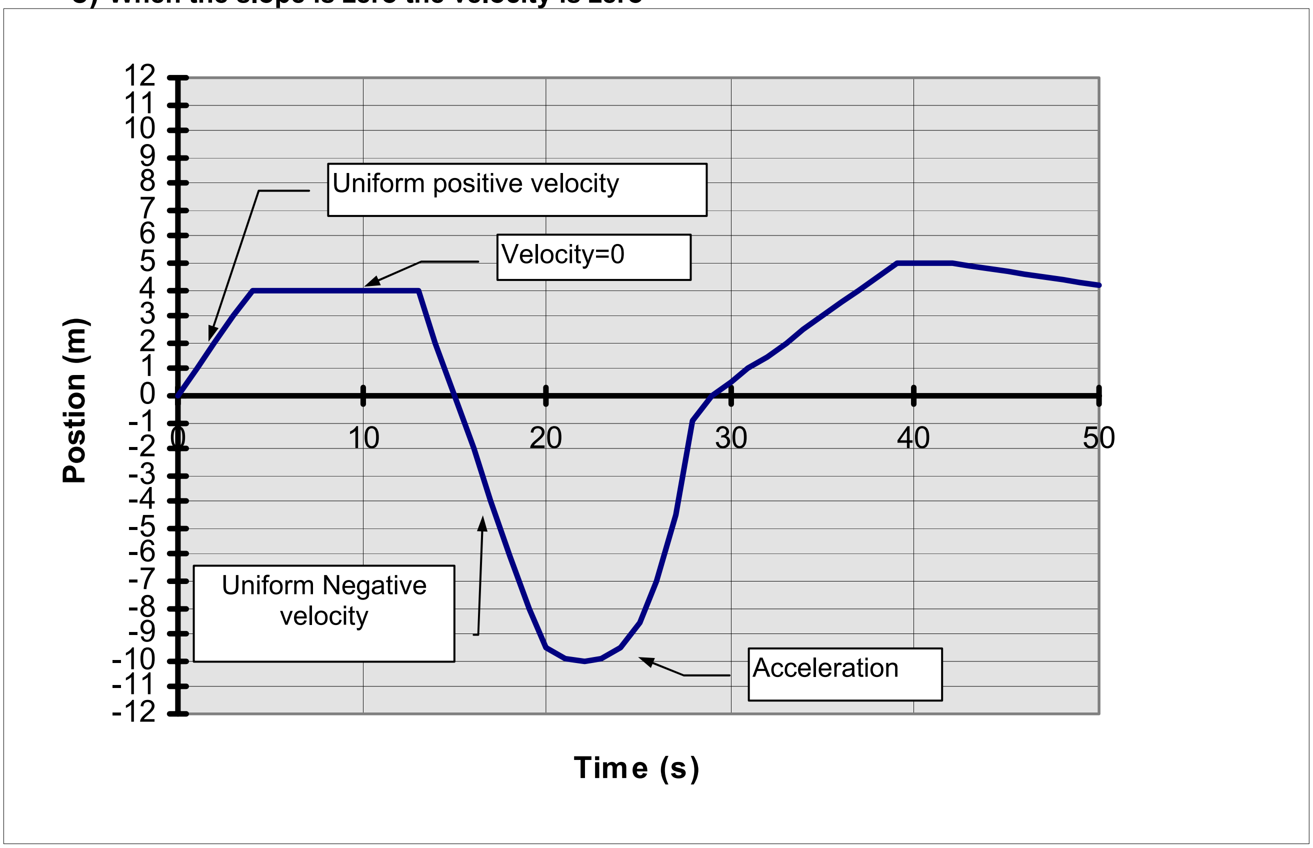


EXERCISE

Position-Time Graphs

Motion can also be represented by a position-time graph. This title implies it's function, it describes an object's position over period of time. The shape of the curve indicates the object's velocity at any given time. Things to know about position time graphs.

1) **Straight lines indicate uniform motion**
2) **Straight lines with positive slope have positive velocity (moving forward)**
3) **Straight lines with negative slope have negative velocity (moving backwards)**
4) **The slope along any point of the curve on a position-time graph gives the velocity of the object**
5) **Curved lines on a position-time plot indicate non uniform motion (acceleration or deceleration or negative acceleration)**
6) **Lines that are curving or starting to curve down and to the right are decelerating**
7) **Lines that are curving or starting to curve up and to the right are accelerating**
8) **When the slope is zero the velocity is zero**



Uniform Acceleration

Uniform acceleration can be defined by the following equation

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \; or \; \vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

this equation is only valid if **acceleration is uniform**. If acceleration is not uniform than this equation gives **average acceleration**

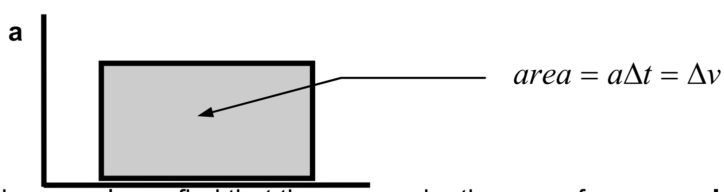
I.e. when entering the on ramp of a highway, a car accelerates from 60km/h [E] to 100km/h [E] in 10s. What is the acceleration?

$$\begin{aligned}
\vec{a} &= \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} \\
&= \frac{100km/h[E] - 60km/h\,[E]}{10s} \\
&= 40\frac{km/h}{s}\,[E](converting\,to\,m/s) \\
&= 11.1\frac{m/s}{s}[E] \\
&= 11.1m/s^2\,[E]
\end{aligned}$$

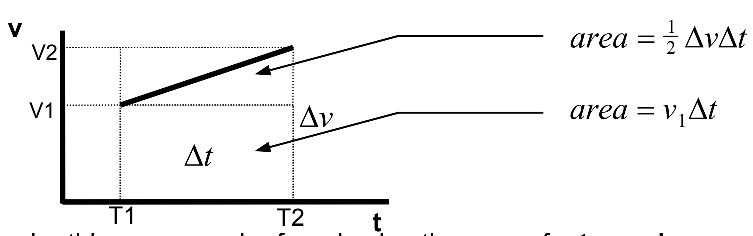
Derived Equations from Graphs

One of the beautiful things about physics is the inner connectivity of concepts. Everything is related to everything else and once you figure out the main concepts, you can figure everything out from there. **Kinematics** is no exception.

i.e.



From this example we find that the area under the curve for an **acceleration-time** graph gives the over all change in velocity. The **velocity-time** graph, on the other hand, gives us a multitude of ways of finding **displacement**.



Area under this curve can be found using the area of a **trapezium**

$$A = \frac{1}{2}(a+b)(h)$$
$$= \frac{1}{2}(v_1 + v_2)(\Delta t)$$
$$= \frac{(v_1 + v_2)}{2}\Delta t$$

$$\boxed{\Delta d = \frac{(v_1 + v_2)}{2}\Delta t \quad or \quad \Delta d = v_{av}\Delta t}$$

The area under this curve can also be found this way ($\text{area} = \text{area of triangle} + \text{area of rectangle}$)

$$\Delta d = v_1 \Delta t + \frac{1}{2}\Delta v \Delta t$$
$$but \quad \Delta v = a\Delta t$$
$$\Delta d = v_1 \Delta t + \frac{1}{2}(a\Delta t)(\Delta t) = v_1 \Delta t + \frac{1}{2}a(\Delta t)^2$$

$$\boxed{\Delta d = v_1 \Delta t + \frac{1}{2}a(\Delta t)^2}$$

The area can also be found be subtracting the area of a triangle from the area of the big rectangle with height $\mathbf{v_2}$ and width Δt

$$\boxed{\Delta d = v_2 \Delta t - \frac{1}{2}a(\Delta t)^2}$$

EXERCISE

Prove that $v_2{}^2 = v_1{}^2 + 2ad$ **from the equation** $v_2 = v_1 + a\Delta t$ (see proof on pg 25)