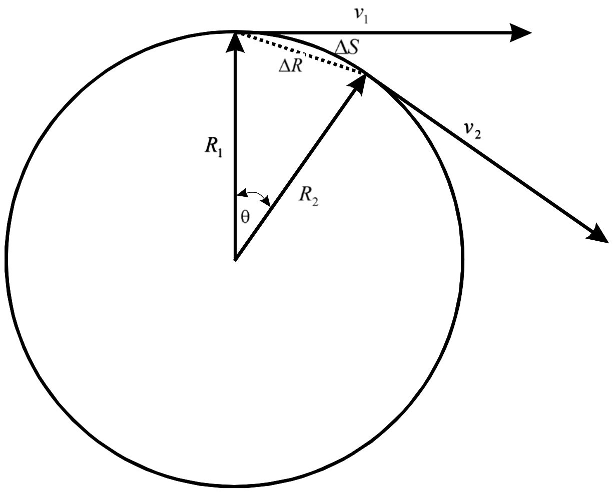
Uniform Circular Motion

One of the most quaint aspects vector motion is the ability to demonstrate acceleration in a situation where speed is constant. Such is the case with circular motion. In this scenario, the object is **speed** is constant but its **velocity** is changing; not in magnitude but in direction. Since the **velocity** is changing. It, therefore, must be accelerating.

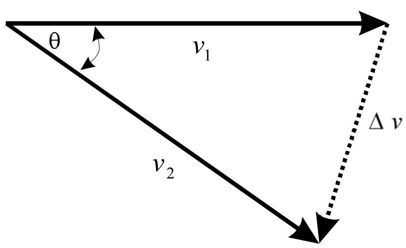


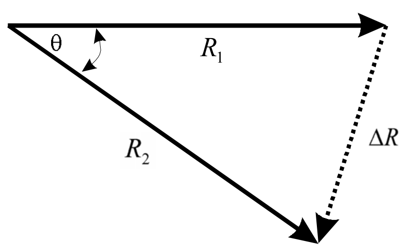
$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

We can find the object acceleration by finding the change in its velocity over time. This becomes a real pain if we have to use vector analysis every time we want to calculate **centripetal acceleration**. Fortunately, there is a clever proof to make these types of calculations really simple.

The change in velocity can be found via vector addition like so.

The neat thing is that a similar vector diagram can be made for the change in the radius vector. The wacky thing is that these two **isosceles triangles** are similar based on the fact that the two equal sides on both triangles are separated by the same angle. If you study the diagram, you can prove this but that's saved for geometry class. The rest of the proof we will work out together right now!





Assumptions: $|\vec{v}_1| = |\vec{v}_2| = v$ and $|\vec{R}_1| = |\vec{R}_2| = R$

$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$ we can eliminate $\Delta\vec{v}$ by using the following relationship $\frac{\Delta v}{v_1} = \frac{\Delta R}{R_1}$

$$\frac{\Delta v}{v} = \frac{\Delta R}{R}$$

$$\Delta v = \frac{v\Delta R}{R}$$

Note: *we are only looking for magnitudes at this point. Direction will be considered next. Hence the removal of vector notation.*

$$a_{av} = \frac{\left(\frac{v\Delta R}{R}\right)}{\Delta t}$$

$$a_{av} = \frac{v\Delta R}{R}\frac{1}{\Delta t}$$

$$a_{av} = \frac{v\Delta R}{R\Delta t}$$

$a_{av} = \frac{v}{R}\frac{\Delta R}{\Delta t}$ but for small angle of θ, $\Delta S \cong \Delta R$

$$a_{inst} = \frac{v}{R}\frac{\Delta S}{\Delta t}$$

$$a_{inst} = \frac{v}{R}v$$

$$a_{inst} = \frac{v^2}{R}$$

$$a_c = \frac{v^2}{R}$$

Where a_c is the centripetal acceleration in m/s^2

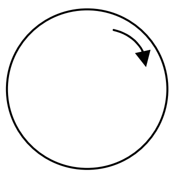
v is the tangential speed of the object in m/s

R is the radius in m

Note: Centripetal acceleration **ALWAYS** point inward toward the center of ration

Equations for Satellites and Rotating Disks

Consider a rotating disk



The linear speed of a point along a rotating disk (at a fixed frequency) is defined as

$v = \dfrac{\Delta d}{\Delta t}$ but $\Delta d = 2\pi R$ (the circumference of the disk) and $\Delta t = T$ (the period of the disk)

therefore $v = \dfrac{2\pi R}{T}$

Now using the centripetal acceleration equation

$a_c = \dfrac{v^2}{R}$ $a_c = \dfrac{\left(\dfrac{2\pi R}{T}\right)^2}{R}$ $a_c = \dfrac{4\pi^2 R^2}{T^2}\dfrac{1}{R}$ $\boxed{a_c = \dfrac{4\pi^2 R}{T^2}}$	But $f = \dfrac{1}{T}$. Therefore $f^2 = \dfrac{1}{T^2}$ $a_c = \dfrac{4\pi^2 R}{T^2}$ $a_c = 4\pi^2 R \times \dfrac{1}{T^2}$ $a_c = 4\pi^2 R \times f^2$ $\boxed{a_c = 4\pi^2 R f^2}$
Where a_c is the centripetal acceleration in m/s^2 T is the period of rotation in s R is the radius in m **Note:** Centripetal acceleration **ALWAYS** point inward toward the center of ration	Where a_c is the centripetal acceleration in m/s^2 f is the frequency of rotation in s R is the radius in m **Note:** Centripetal acceleration **ALWAYS** point inward toward the center of ration