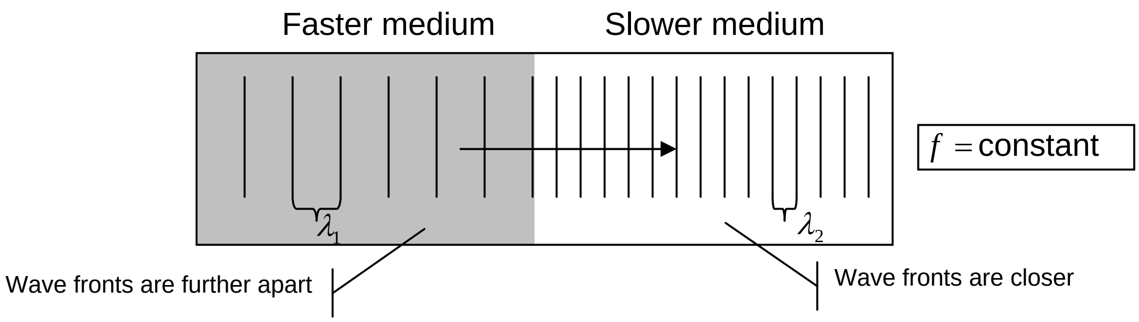
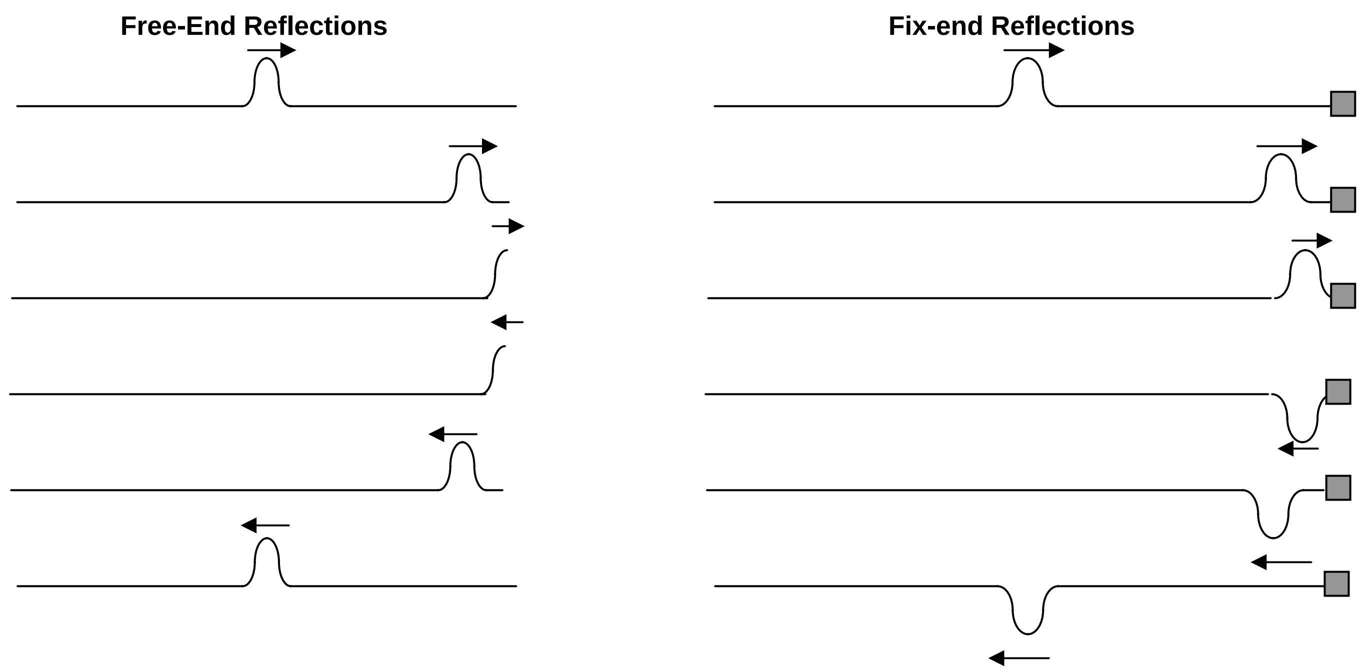
Waves traveling travelling from one medium to another will exhibit different characteristics within each medium.

Rules

- A wave of fixed frequency will have a shorter wavelength when passing from a fast medium to a slower medium. The diagram below represents the crests of water waves moving from deep water to shallow water – top view.



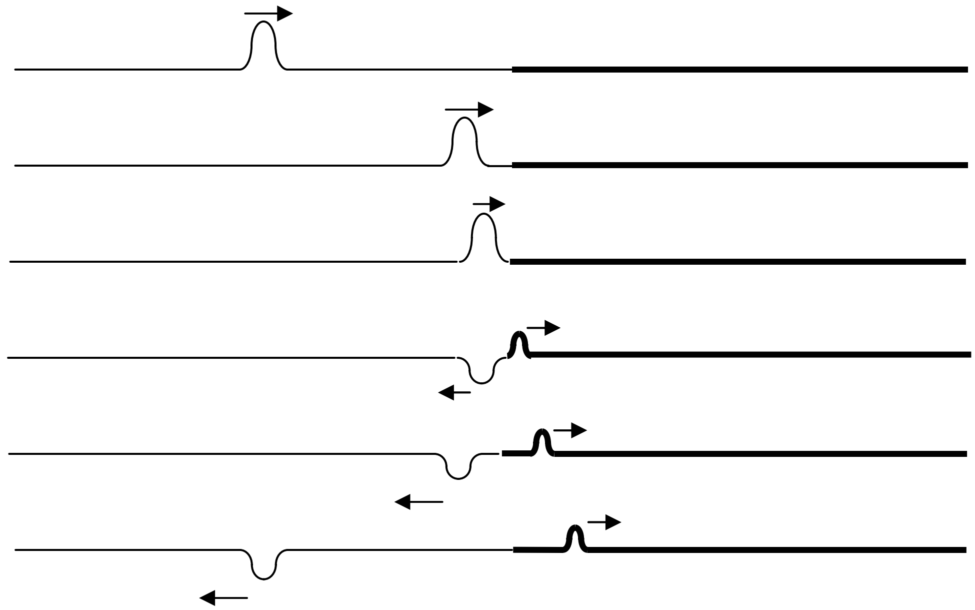
- Waves will reflect at the boundary of a medium in one of two ways.



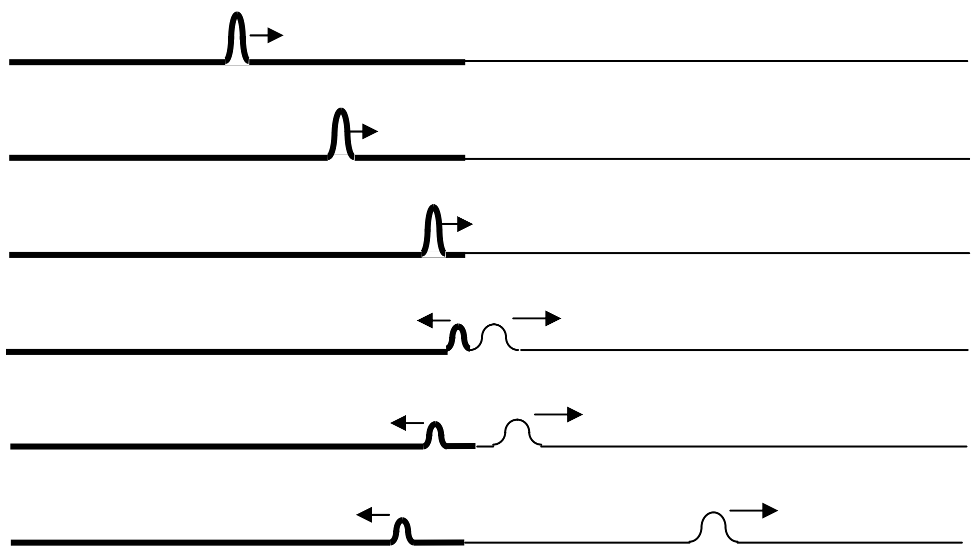
Free-end Reflections: Can be represented by water waves reflecting against a break-wall.

Fixed-end Reflections: Can be represented by waves traveling through a guitar string.

- Waves moving from a **fast** medium to a **slow** medium will experience a reflection as illustrated below. The pulse traveling from the faster medium sees the boundary as a fixed end and reflects accordingly.



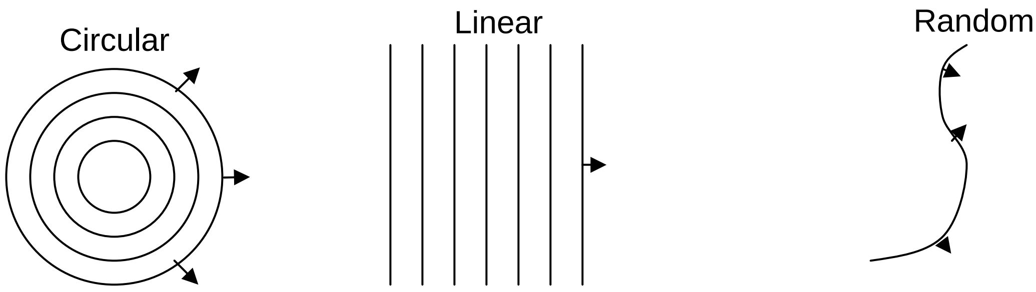
- Waves moving from a **slow** medium to a **fast** medium will experience a reflection as illustrated below. The pulse traveling from the faster medium sees the boundary as a free end and reflects accordingly.



Two Dimensional Waves

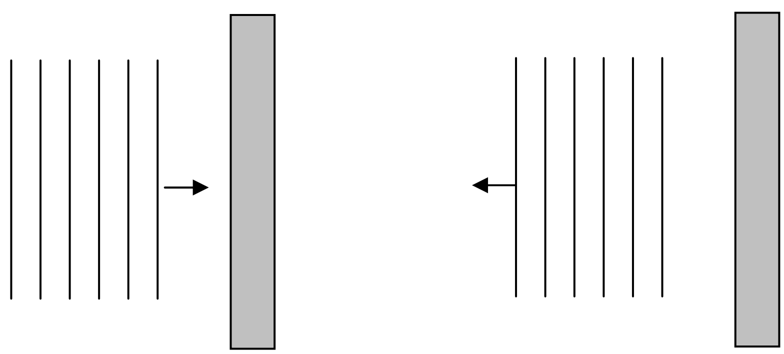
- Waves experience ***rectilinear propagation*** (transmission in a straight line) provided the wave does not encounter a barrier or move into another medium.

- **Types of fronts.**

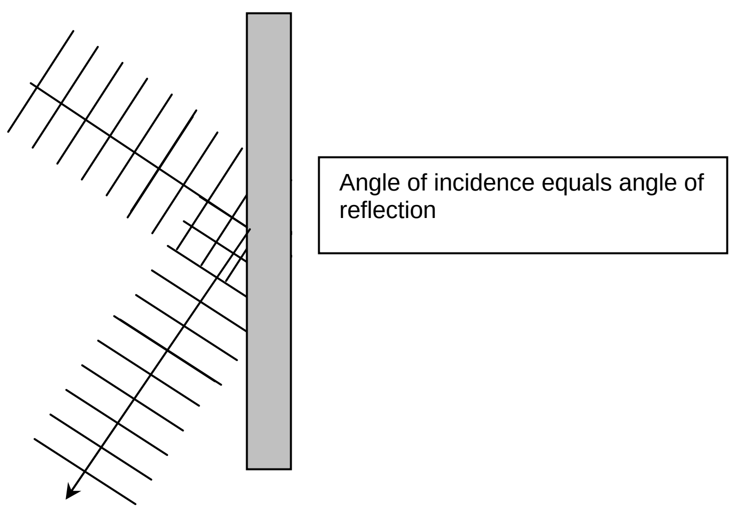


- **Reflection of two dimensional waves**

Straight on Collision



Off-Angle Collision



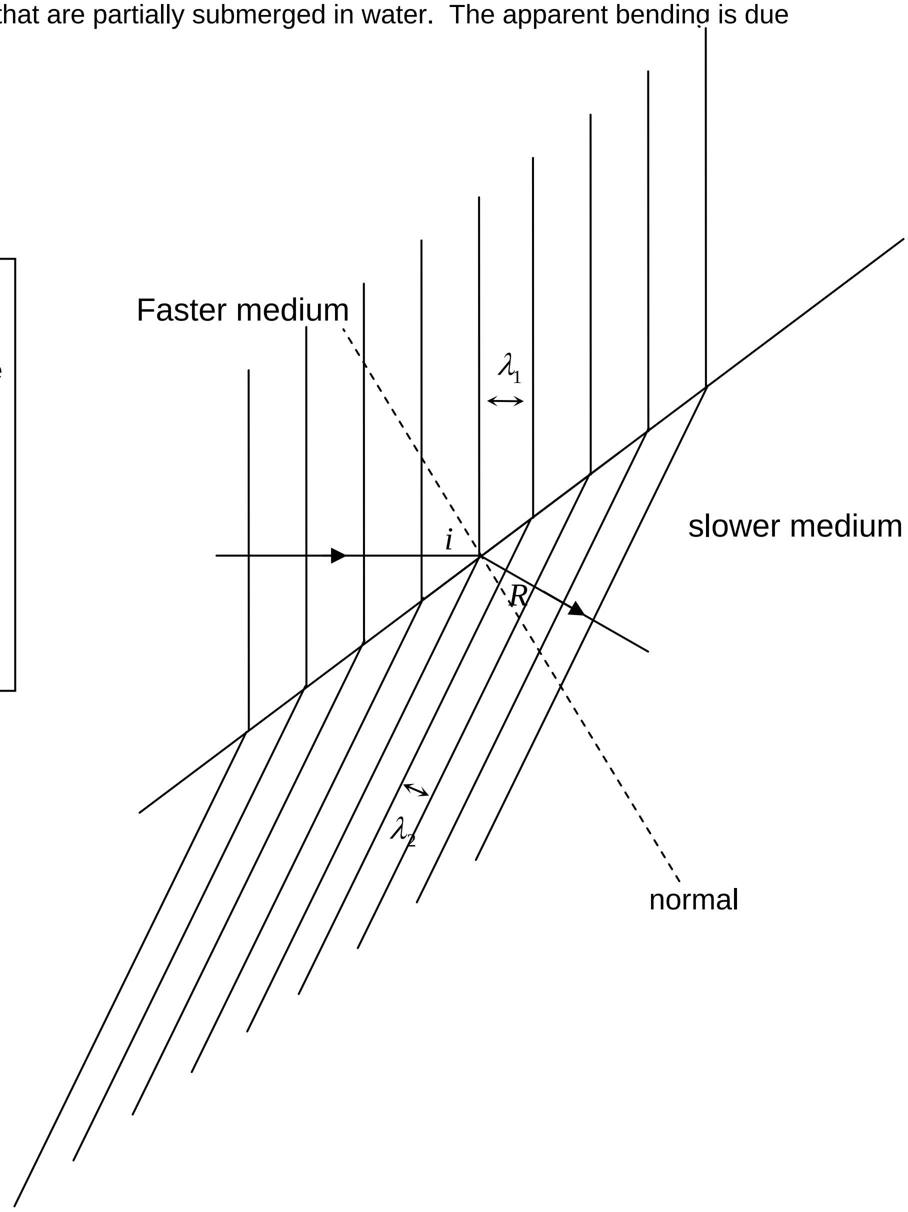
Refraction of Waves

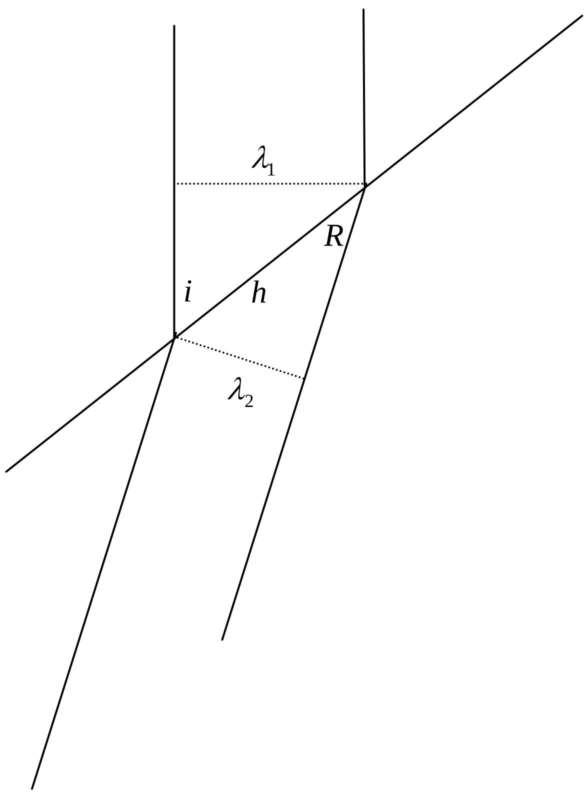
When a wave travels from a one medium to another, the wave will experience **refraction**. Refractions is the bending of a wave away or toward the perpendicular line that defines the boundary between the two medium. For example: the distortion of the image of objects that are partially submerged in water. The apparent bending is due to ***refraction***.

i is the angle of incidence
R is the angle of refraction

Note: The angle of incidence and refraction is angle measured between **the wave** ray (direction vector) and the normal.

Upon closer analysis, you will also note that the angle of in incidence and refraction can be measured between the boundary line and the wave fronts themselves as illustrated in the next diagram





Snell's Law

$\sin i = \frac{\lambda_1}{h}$ and $\sin R = \frac{\lambda_2}{h}$ rearranging $h = \frac{\lambda_1}{\sin i}$ and $h = \frac{\lambda_2}{\sin R}$

$$\frac{\lambda_1}{\sin i} = \frac{\lambda_2}{\sin R}$$

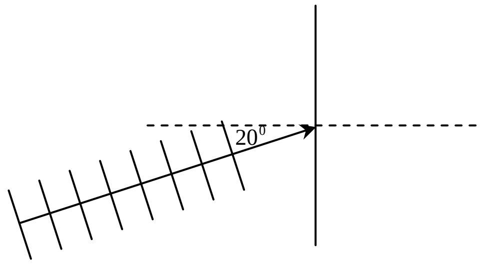
$$\frac{\lambda_1}{\lambda_2} = \frac{\sin i}{\sin R}$$

$$\boxed{\frac{\sin i}{\sin R} = \frac{\lambda_1}{\lambda_2}}$$

but $v = f\lambda$ so $\frac{v_1}{v_2} = \frac{f\lambda_1}{f\lambda_2}$ Therefore $\boxed{\frac{\sin i}{\sin R} = \frac{v_1}{v_2}}$

Example: A wave of wavelength of 2m is travelling through a medium at 2.0m/s. The wave then moves into a second medium where the wave speed is reduced to 1.0m/s. The wave enters a the medium at an angle of 20^0 to the normal,

a) What is the angle of refraction?
b) What is the wavelength of the wave in the second medium?



Dispersion

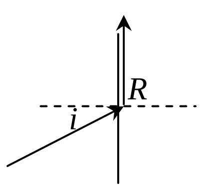
As discussed earlier, waves of the same frequency travel at different velocities in different media. It is assumed however that the wave velocity is constant for all frequencies within the same medium. This however is not entirely accurate. There can be a slight difference in wave speed within the same medium, for different frequencies. There are some very concrete and observable examples of this phenomenon.

1) The Prism: The prism is an example of dispersion with light. As you know when light enters a prism, it is split into its components, rendering the full spectrum from red to violet. This is caused by a slight discrepancy in the speed of light, within glass. Violet light travels more slowly than red light. As a result, violet light will refract more that red light, rendering the characteristic spectrum.

2) Thunder: During a thunderstorm, the effect of dispersion can be best observed for lighting strikes that are some distance away. It is not however the light that experiences the most dramatic effects of dispersion, it is the sound. Typically when lightning strikes, the crack of thunder is heard a few moments later, soon followed by a deep rumble. This difference in the sound isn't heard for very close strikes. Those produce a sharp, and violent explosive sound. The reason for the difference is that speed of sound is dependent on both frequency and sound intensity. Thunder is an explosion, essentially causing a major disruption in the medium. Under these conditions, higher frequencies travel more quickly than lower frequencies, hence the "crack" followed by the "rumble".

Total Internal Reflection

Recalling from last years, waves traveling from a slower medium to a faster medium will experience total internal reflection. Total internal reflection occurs when the critical angle is exceeded. The critical angle is defined as the incident angle that results in angle of refraction of 90^0

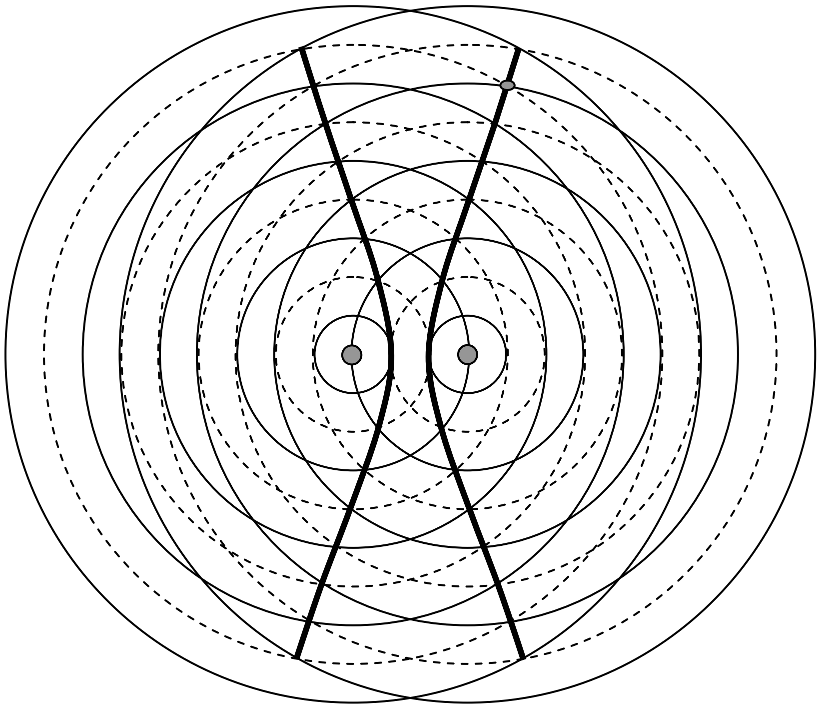


$$\frac{\sin i}{\sin 90} = \frac{v_1}{v_2} \qquad \boxed{\therefore \sin i = \frac{v_1}{v_2}} \quad \text{or} \quad \boxed{\sin i = \frac{\lambda_1}{\lambda_2}}$$

Examples:

1) A wave traveling at a speed of 40 m/s enters a second medium. If the speed in the second medium is 65m/s, find the critical angle.
2) Find the critical angle between two mediums if the wavelength of any given wave is increased by 40% when traveling in the faster medium.

Interference Pattern From Two Point Sources



—— Represents the crests

- - - - Represents the troughs

The diagram represents two point sources generating waves in phase with each other. The distance between two consecutive crests or troughs is one wavelength. The two sources are set one wavelength apart. Where the crests and troughs meet, destructive interference occurs, resulting in nodal lines that take the shape of a hyperbola.

In this example, only a few nodal lines will be produced. The number of nodal lines will increase when:

a) The frequency increase –separation of point sources remains constant
b) The frequency remains constant – the separation of the point sources increases

This phenomenon can be predicted easily by making the following observation. Pick any point along the nodal lines and count the number of wavelengths that separate each point source from that point.

Take for instance the point on the nodal line to the right. This point is 4λ away but the point on the right is 4.5λ away. Therefore the path difference is

$$|4\lambda - 4.5\lambda| = \tfrac{1}{2}\lambda$$

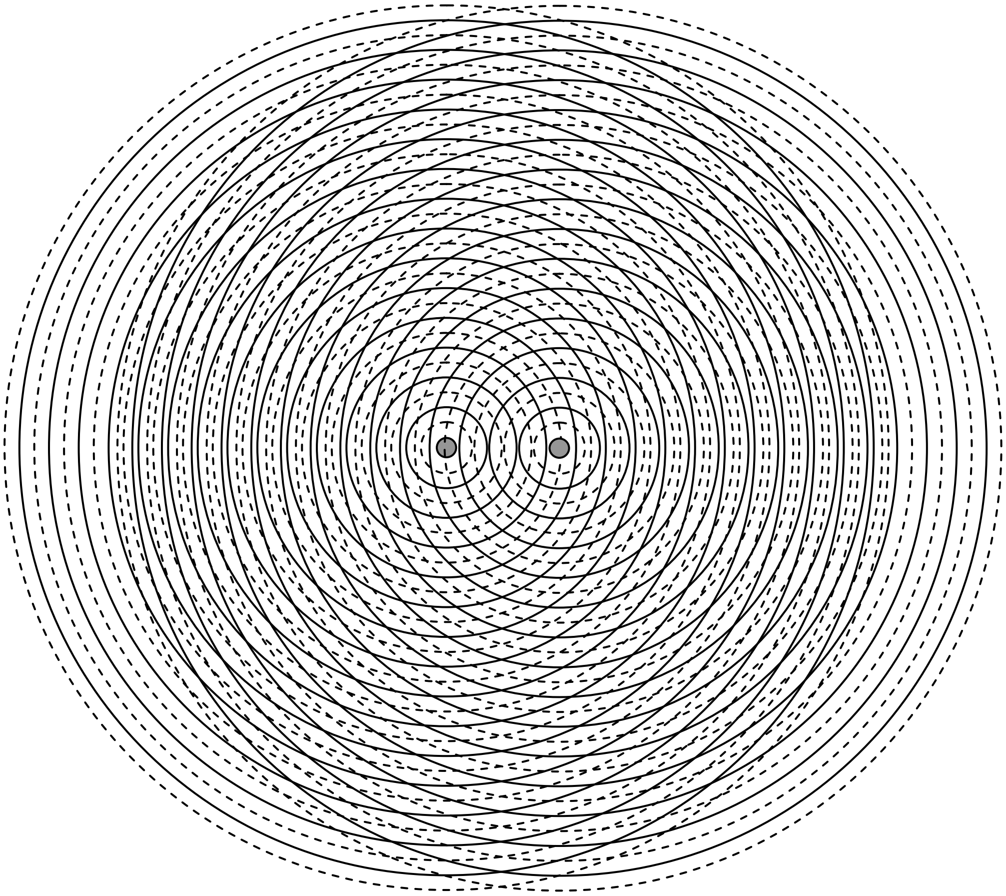
A path difference of $\frac{1}{2}\lambda$ results in destructive interference. This will occur any time the waves are out of phase (phase angle= π)

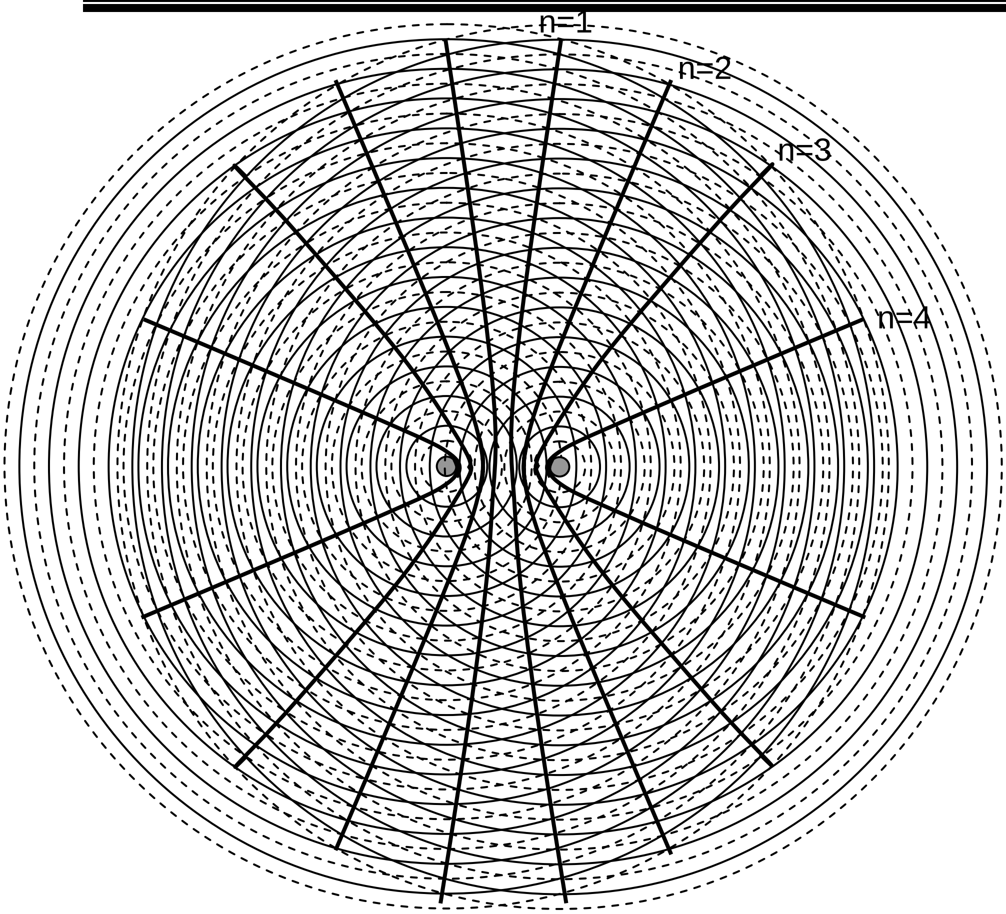
In general, $|Path\ difference| = (n - \tfrac{1}{2})\lambda$

Using the diagram on the next page we are going to prove this relationship

Procedure:

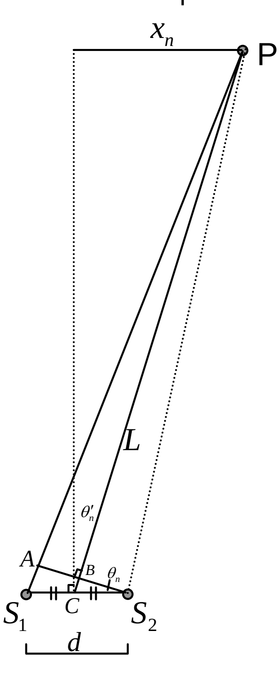
1. Draw all the nodal lines on the diagram and label them n=1,n=2, etc. Since the pattern will be symmetrical, there will be two n=1 and two n=2.
2. Determine the path difference in wavelengths and attempt to prove the formula.





This interference pattern generates 8 nodal lines, or 4 symmetrical pairs. The path difference between the points located on the nodal lines and the two sources are 0.5λ, 1.5λ, 2.5λ and, 3.5λ for nodal lines 1,2,3 and 4 respectively.

The above demonstration of this physical phenomenon does actually lead to an important set of formulae that are quite important in optics. As mentioned the earlier, any point on any nodal line will be exactly $\left(n-\frac{1}{2}\right)\lambda$ apart. The problem with the above analysis is that it is very difficult to measure these path differences accurately. Therefore another procedure is necessary. Consider the following situation.



To complete the analysis of this question requires a few assumptions that will only work when P is very far away from the point sources compared to their separation. The reason for this is the assumption that must be made about $\angle S_1AS_2$. We are going to use two relationships.

$$\sin\theta_n = \frac{AS_1}{d} \text{ and } \sin\theta_n' = \frac{x_n}{L}$$

There is one small problem with our assumption about $\sin\theta_n = \frac{AS_1}{d}$, ΔS_1AS_2 is not a right angle triangle technically. So the sine relationship really doesn't apply unless we assume P is very far away. The sides of the isosceles triangle, PAB, approximately become parallel when P is very large compared to d. If we make this assumption then $\angle S_1AS_2 \cong 90^0$, allowing us to treat the triangle like a right angle triangle.

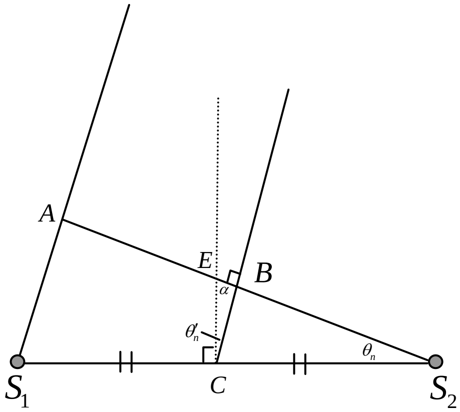
let's first consider $\sin\theta_n = \frac{AS_1}{d}$

$d\sin\theta_n = AS_1$ but AS_1 is the path difference between P S_1 and P S_2. Recalling earlier

$$|path\ difference| = \left(n-\tfrac{1}{2}\right)\lambda \qquad \therefore d\sin\theta_n = \left(n-\tfrac{1}{2}\right)\lambda$$

or

$$\sin\theta_n = \left(n-\tfrac{1}{2}\right)\frac{\lambda}{d}$$



ΔS_2EC	ΔCEB
$\theta_n + \alpha + 90^0 = 180^0$	$\theta_n' + \alpha + 90^0 = 180^0$

$$\therefore \theta_n = \theta_n'$$

Now consider the formula $\sin\theta_n = \left(n - \tfrac{1}{2}\right)\dfrac{\lambda}{d}$

Therefore $\sin\theta_n' = \left(n - \tfrac{1}{2}\right)\dfrac{\lambda}{d}$

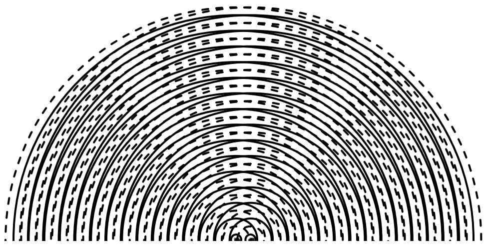
$$\frac{x_n}{L} = \left(n - \tfrac{1}{2}\right)\frac{\lambda}{d}$$

$$\lambda = \frac{x_n}{L}\frac{d}{\left(n - \tfrac{1}{2}\right)}$$

Examples:

1. Two point sources are separted by distance of 2.0cm. You pick a point along the first nodal line. The point is 1.0 m away from the midpoint between the two point sources. The point is also 10cm away from the perpendicular bisector that separted the two point sources. Find the wavelength of the wave.

2. Find the wavelength of the interfering waves using the above formulae and aforementioned techniques.

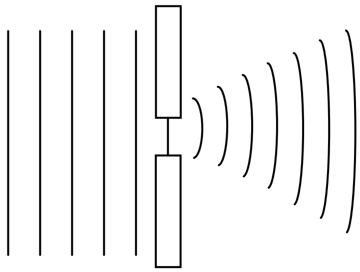


Diffraction

We have previously discussed wave behaviour in terms of reflection and refraction. Waves also experience a phenomenon called **diffraction**. Diffraction is the ability for waves to bend around barriers or to expand out from small openings. It's diffraction that enables us to hear sound around corners for example.

Diffraction and Path Restrictions

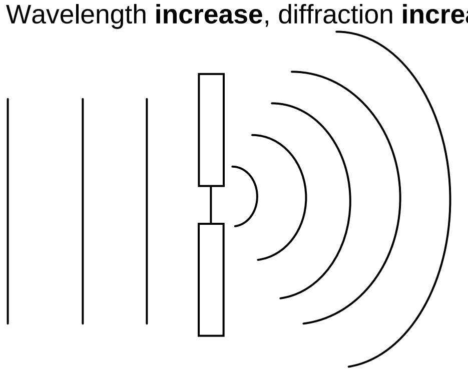
Waves passing through small openings in barriers produce an interesting yet unexpected effect. Straight waves entering a small opening will become curved when they emerge on the other side.



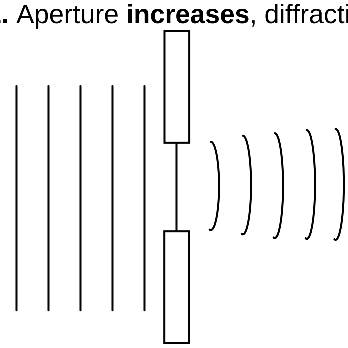
Diffraction is dependent on both wavelength and aperture size.

Rules:

1. Wavelength **increase**, diffraction **increases**



2. Aperture **increases**, diffraction **decreases**



For now, it suffices to simply describe the diffractive behaviour of waves. In the next section we will discuss the theories that attempt to explain diffractions.