Waves

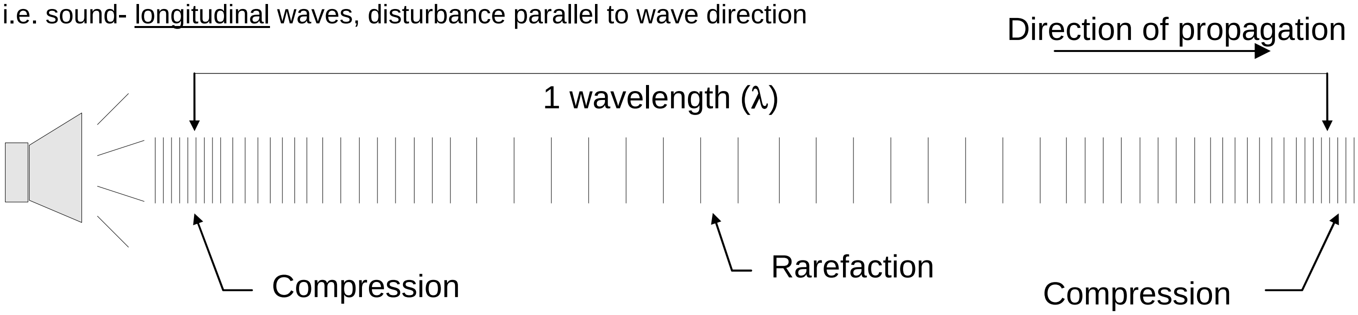
Wave: is a transfer of energy as a disturbance. The medium itself, however, does not move.

Three types of waves

1. **Mechanical waves**: These are waves that are governed by Newton's laws. Examples are sound, seismic, water waves. These types of waves require a medium and can be longitudinal, transversal or tortional (rotational)
2. **Electromagnetic waves:** These are waves do not require a medium. These types of waves have both a magnetic and electrical component to them. Electromagnetic waves are always transversal and travel at or near the speed of light.
3. **Matter waves:** Refer to the behaviour of many subatomic particles, such as electrons, protons, neutrons, etc., which at times behave like waves at other times like particles.

Longitudinal Waves

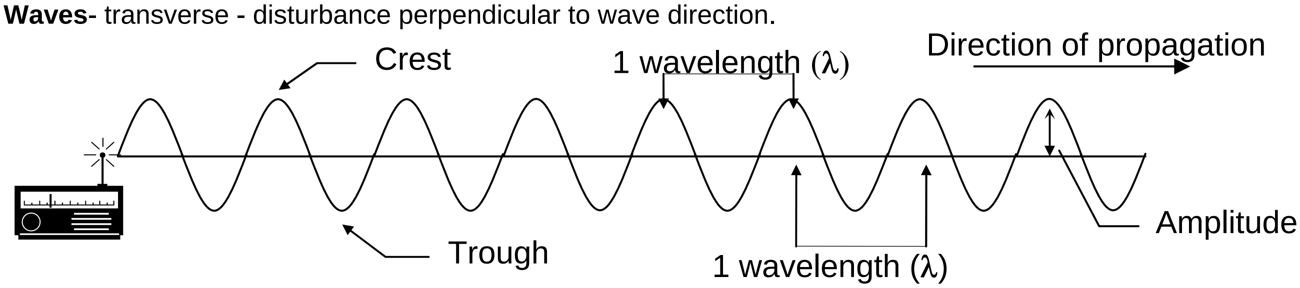
i.e. sound- longitudinal waves, disturbance parallel to wave direction



The **amplitude** of a longitudinal wave is difficult to observe at times. In the above example, the amplitude would be the measure of how far a region of air is displaced from its equilibrium position. This is usually measured by using a microphone. The amount to which the diaphragm of the mic is force to move is a measure of amplitude. Another example would be the observable behaviour of the sub woofer. At high sound level, the speaker is pushed outward and inward. The distance to which the speaker deviates from the equilibrium position (volume =zero) is a measure of the amplitude.

Transverse Waves

Radio Waves- transverse - disturbance perpendicular to wave direction.



The amplitude of the wave is perpendicular to the motion of the wave. In the case of an electromagnetic wave, there are two components. In the above example only shows one component, either the electrical or magnetic component. The second component of the wave would appear to be coming in and out of the page.

The wave equations

1) $f = \frac{cycles}{time}$ is basic frequency equation which defines the Hertz (Hz) as the number of cycles, or oscillations, per second.

2) $f = \frac{1}{T}$ which can be re-written as $T = \frac{1}{f}$, where T is the period in seconds and f is the frequency in Hertz (Hz).

3) $v = f\lambda$ where v is the velocity in m/s, f is the frequency in Hz and λ is the wave length in meters.

example 1 – A wheel rotates 120 times in 30s.

$f = \frac{cycles}{time} = \frac{120}{30} = 4Hz$ yes…very easy!

example 2 – a radio wave has a wavelength of 2.887m. What is its frequency? Note, radio waves travel at the speed of light 3.0x10^{8}m/s.

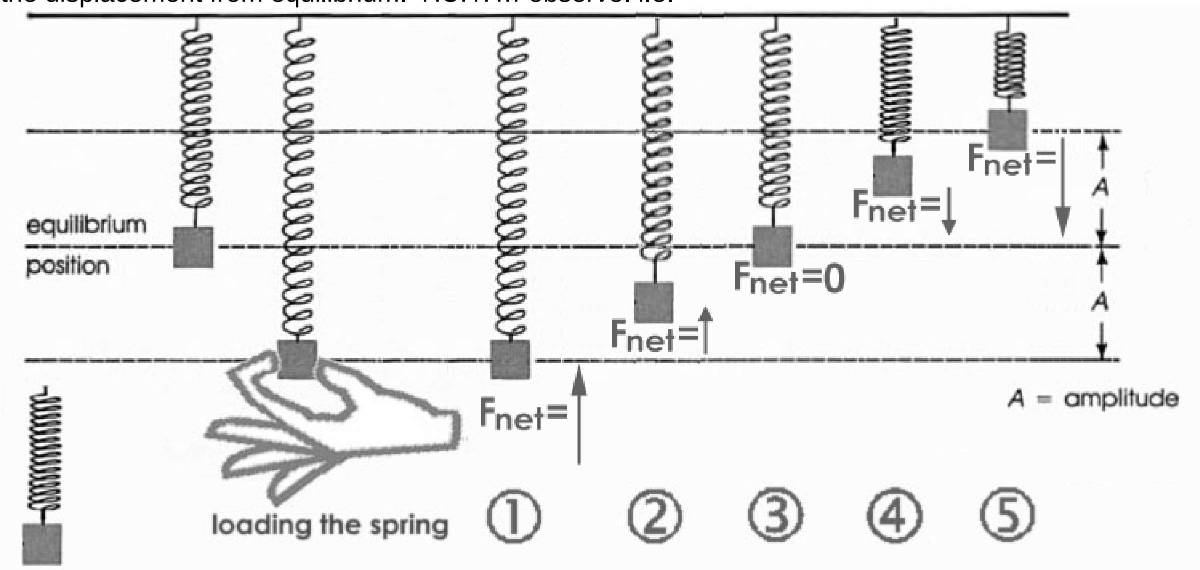
$v = f\lambda$

$f = \frac{v}{\lambda}$

$f = \frac{v}{\lambda} = \frac{3.00 \times 10^8}{2.887} = 1.039 \times 10^8\ Hz = 103.9 \times 10^6\ Hz = 103.9\ MHz$ (or more commonly known as *"The Hawk"*.)

Special Oscillations
Simple Harmonic Motion

Simple Harmonic Motion (SHM) – is achieved when the restoring force in an oscillating system is proportional to the displacement from equilibrium. HUH?… observe. i.e.



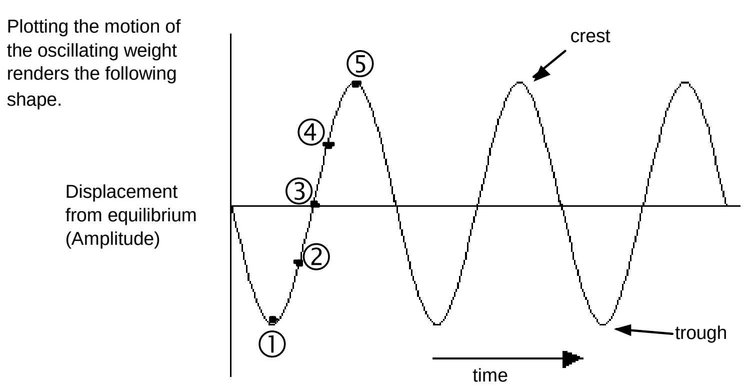
1. At this point you get max force up (↑) because spring is at its maximum extension. Therefore, max vertical acceleration.

2. The spring is still exerting a Net upward force (↑) but with less strength as the spring recoils. (i.e. $F = kx$)

3. The upward force of the spring equals the downward force of gravity. $F_{net} = 0$, $a = 0$ but **velocity** of mass is at its **maximum** and in an upward direction.

4. The upward force of the spring is less than that of the downward force of gravity. Therefore the net force is downward (↓). Acceleration is now negative, therefore the mass slows down.

5. Minimum vertical force from spring at this point. Largest force in the system is F_g. Therefore the net force is downward (↓) is at its maximum in the downward direction.

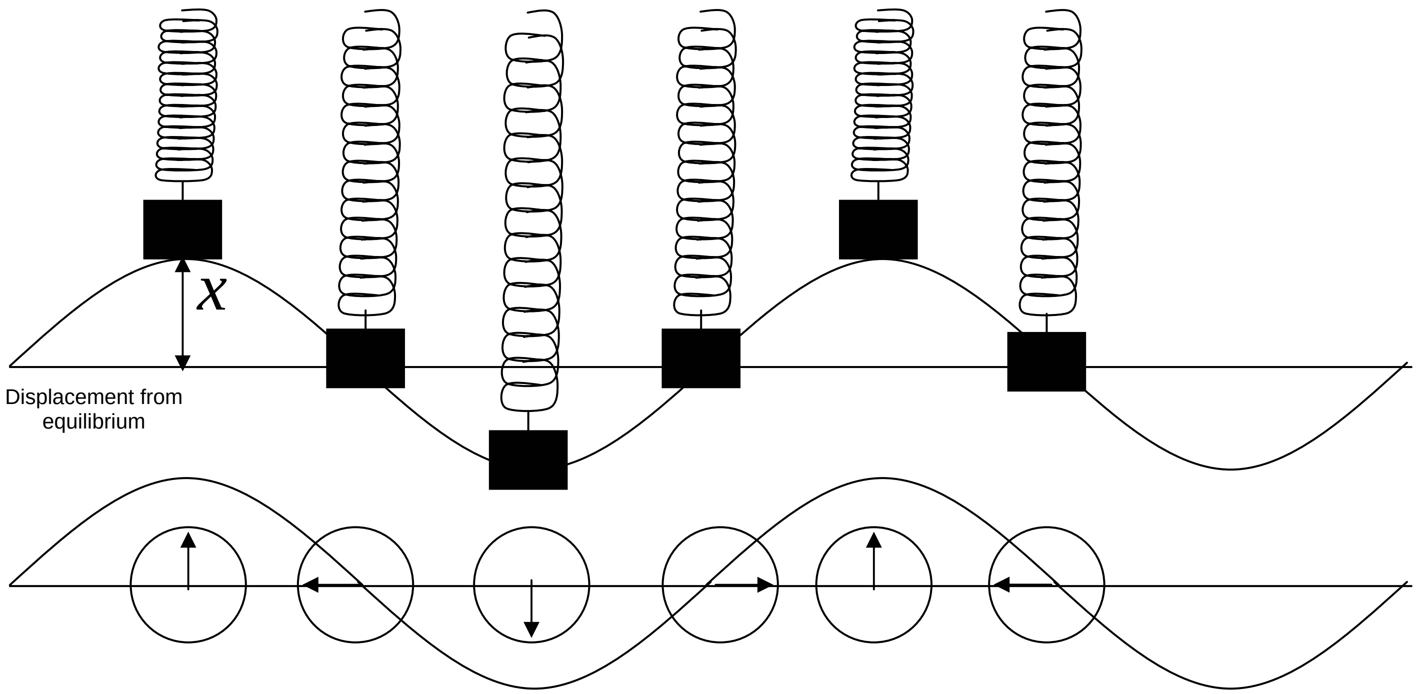


The **sine curve**!… well sort of… this one is significantly out of phase but you get the idea, its the overall shape that counts (for the sake of clarity, you could argue that this is also a cosine wave as well. Please note

$\sin\theta = \cos(\theta - \frac{\pi}{2})$). The behaviour of the above curve (previous page) begins to resemble a sine curve at ③.

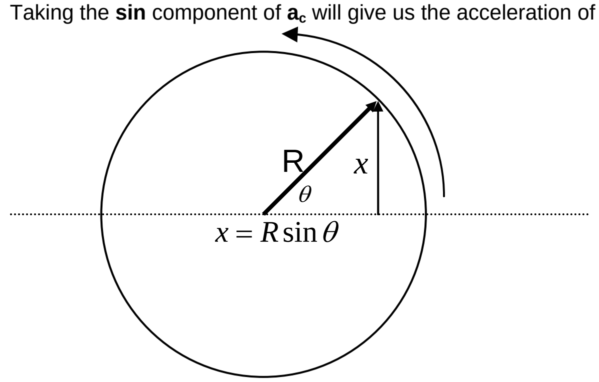
The **restoring force** is proportional to the displacement from equilibrium. In other words, the **net force** exerted by the spring and gravity on the weight is greatest at the **crests** and **troughs**; these crests and troughs are also where maximum **acceleration** occurs. The **net force** is ZERO as the weight passes through equilibrium (amplitude = 0), therefore acceleration is also zero through this point.

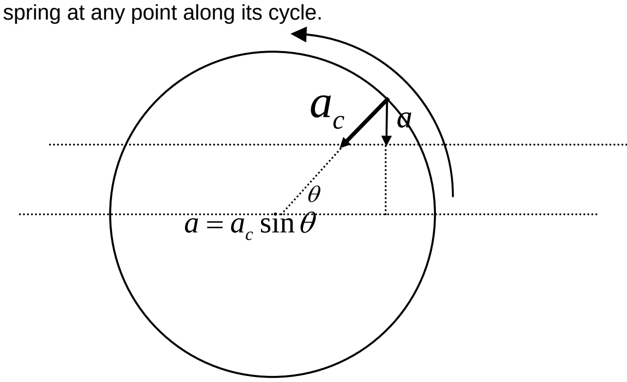
The derivations for the following formulae can be found on [pg 449-451 (Fundamentals of Physics)]. The key to understanding the derivation for these formulae is to understand that the oscillating spring **oscillates** in the pattern of a sine curve.



$\sin(90^\circ) = 1$ $\sin(180^\circ) = 0$ $\sin(270^\circ) = -1$ $\sin(0^\circ) = 0$ $\sin(90^\circ) = 1$ $\sin(180^\circ) = 0$

Notice that the **displacement** of the oscillating spring overtime can be represented by the **sin** components of any rotating disk of radius **R**. A similar argument can be made for the **acceleration** components. Since the disk is rotating, the disk will experience **centripetal acceleration**. Taking the **sin** component of $\mathbf{a_c}$ will give us the acceleration of the oscillating spring at any point along its cycle.





Now consider the above two equations for each circle, $x = R\sin\theta$ and $a = a_c \sin\theta$. Taking the ratio of the two equations yields the following relationship

$$\frac{x}{a} = \frac{R\sin\theta}{a_c \sin\theta}$$

$$\frac{x}{a} = \frac{R}{a_c}$$

Since R and a_c are both constants for a uniformly rotating disk, therefore the ratio $\frac{a}{x}$ must be a constant. This implies that $a \alpha x$. Specifically, **the acceleration of the mass oscillating on a spring increases as displacement from equilibrium increases**.

From previous sections we know that $a_c = \frac{4\pi^2 R}{T^2}$ substituting for a_c in the above equation we get

$$\frac{x}{a} = \frac{R}{\frac{4\pi^2 R}{T^2}}$$

$$\frac{x}{a} = \frac{T^2}{4\pi^2}$$

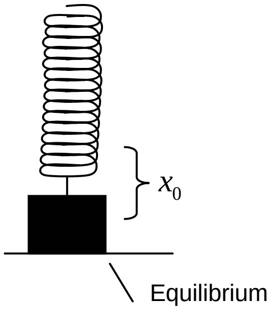
$$T = \sqrt{\frac{4\pi^2 x}{a}}$$

$$T = 2\pi\sqrt{\frac{x}{a}}$$

Where x is displacement from equilibrium in m, a is the acceleration of the oscillating object in m/s^2 and T is the period in seconds.

Period of a Mass Hanging From a Spring

A suspended mass, at rest, is equilibrium. This implies that $F_{net} = 0$, further implying that $|F_g| = |F_x|$. In this state, the spring is experiencing extension. Any further change in x_0 will result a net force upward or downward.



Therefore, F_{net} is only dependent on the deviation from the equilibrium position. Therefore $F_{net} = kx$ or $ma = kx$. How does one justify this? Ignore the fact that spring is already under extension and assume the value x is the extension of the spring. This actually can be proven mathematically to be true by the following argument. Note, in this example x_0 and x represent the initial extension and additional extension respectively.

$$F_{net} = F_x - F_g$$

$$ma = k(x + x_0) - F_g$$

$$ma = kx + kx_0 - F_g$$

remember $kx_0 - F_g$ cancel to zero because those are the forces at equilibrium

$$ma = kx + 0$$

$$ma = kx$$

$$\frac{m}{k} = \frac{x}{a}$$

Substituting in for $\frac{x}{a}$ with $\frac{m}{k}$

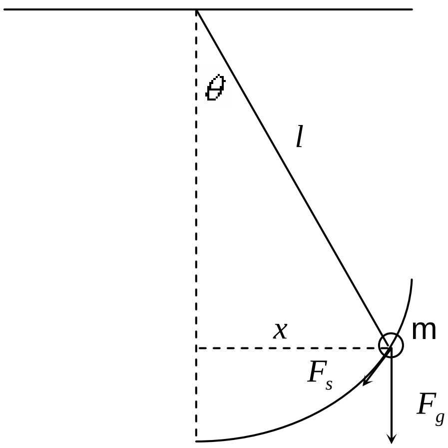
$$T = 2\pi\sqrt{\frac{x}{a}}$$

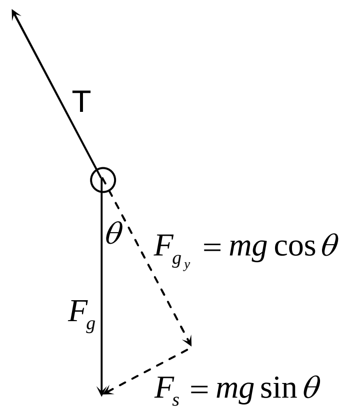
$$T = 2\pi\sqrt{\frac{m}{k}}$$

Period and The Simple Pendulum

The simple pendulum exhibits simple harmonic motion for small values of θ. In the diagram below x represents the position from equilibrium, l represents the length of the pendulum, F_s represents the restoring forces, which is the component of gravity along the arc, F_{g_y} represents the component of gravity that counters the tension T and m represents the mass.

A technical point, the restoring force F_s is **not** along x which represents the amplitude. The definition of SHM (Simple Harmonic Motion) is that the restoring force is directly proportional the amplitude represented by x. Therefore the simple pendulum will only exhibit SHM for small values θ because x and the arc length, for small values of θ, are nearly the same.





Two relationships

$\sin\theta = \frac{x}{l}$ and $\sin\theta = \frac{F_s}{F_g}$

$$\frac{x}{l} = \frac{F_s}{F_g}$$

$$\frac{x}{l} = \frac{F_s}{mg}$$

$$\frac{x}{l} = \frac{F_s}{m}\frac{1}{g}$$

$$\frac{x}{l} = a_s\frac{1}{g} \qquad (\therefore F_s = ma_s)$$

$$\frac{x}{l} = \frac{a_s}{g}$$

$$\frac{x}{a_s} = \frac{l}{g}$$

$$T = 2\pi\sqrt{\frac{x}{a_s}}$$

$$\therefore T = 2\pi\sqrt{\frac{l}{g}}$$

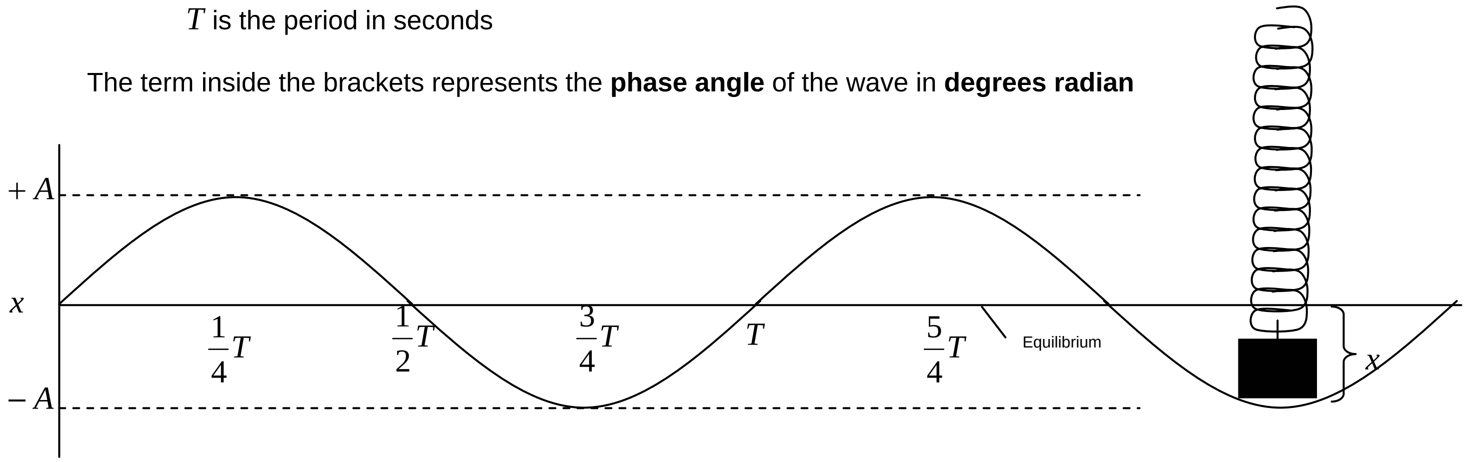
SHM, Sine Waves and Phase Angles

Waves exhibiting simple harmonic motion are sinusoidal and follow the general wave equation of

$$x = A\sin\left(\frac{2\pi t}{T}\right)$$

Where x is the amplitude of the wave at any given time t
A is the maximum amplitude in m
t is the time in seconds
T is the period in seconds

The term inside the brackets represents the **phase angle** of the wave in **degrees radian**



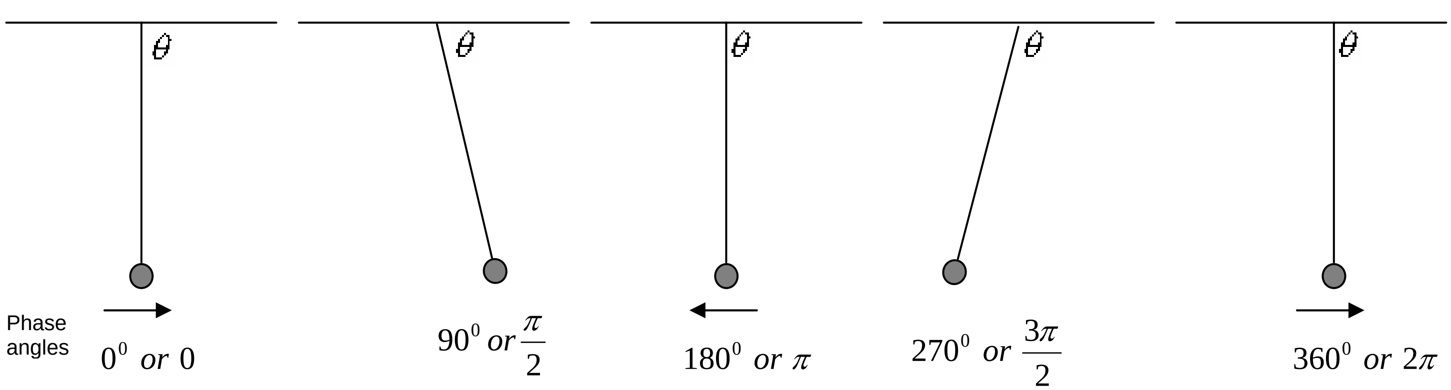
At $t = \frac{1}{4}T$, phase angle = $\frac{\pi}{2}$ (note: $\sin\left(\frac{\pi}{2}\right) = 1$)

At $t = \frac{1}{2}T$, phase angle = π (note: $\sin(\pi) = 0$)

At $t = \frac{3}{4}T$, phase angle = $\frac{3\pi}{2}$ (note: $\sin\left(\frac{3\pi}{2}\right) = -1$)

At $t = T$, phase angle = 2π (note: $\sin(2\pi) = 0$)

The concept of the **phase angle** has absolutely nothing to do with the physical angle that the object experiencing SHM might find itself in… ok that's vague… here is an example.

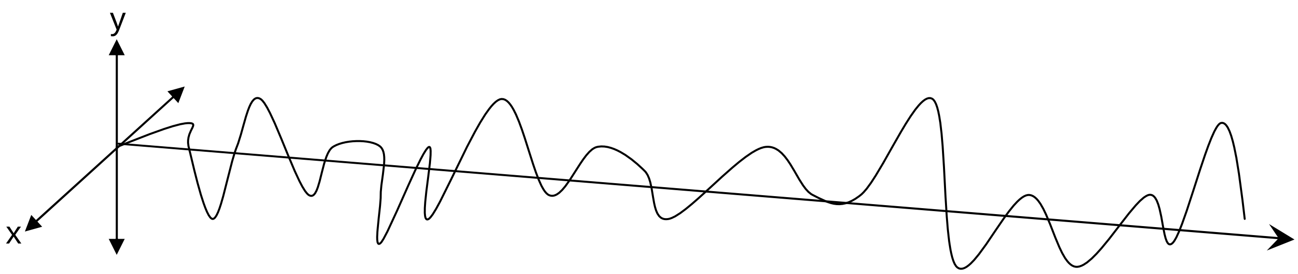


The phase angle has absolutely nothing to do with θ, although certain values of θ correspond to positions within the cycle of the SHM.

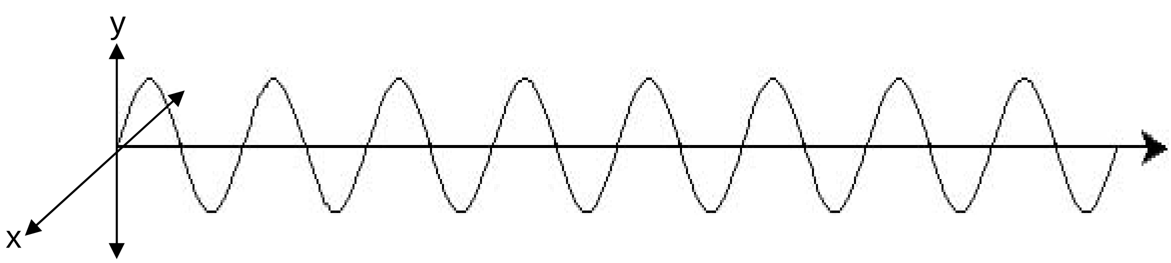
Polarization

Transverse waves, such as radio waves and light, can vibrate in a number of planes at any given time. Any given electromagnetic wave can vibrate in a vertical plane, a horizontal plane, or any plane in between.

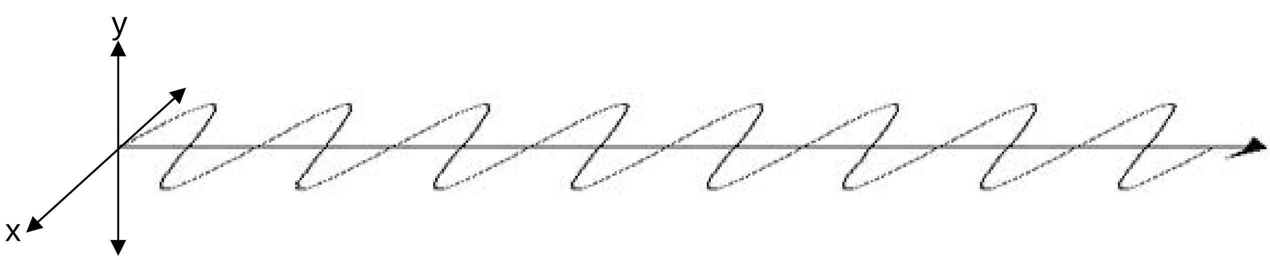
Random Polarization: The transverse waves are oscillating in several planes at any given time. This is caused by several sources generating waves at the same time. The resultant wave is chaotic in appearance and does not resemble a simple sine wave.



Vertical polarization: Waves only vibrate in the vertical plane.

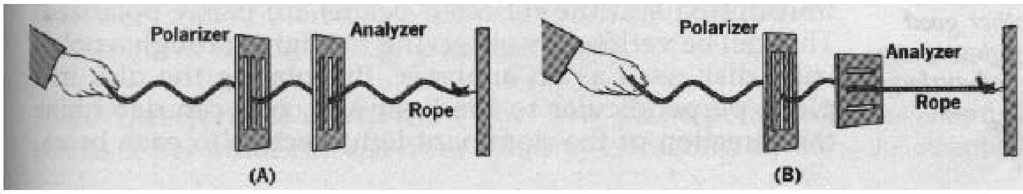


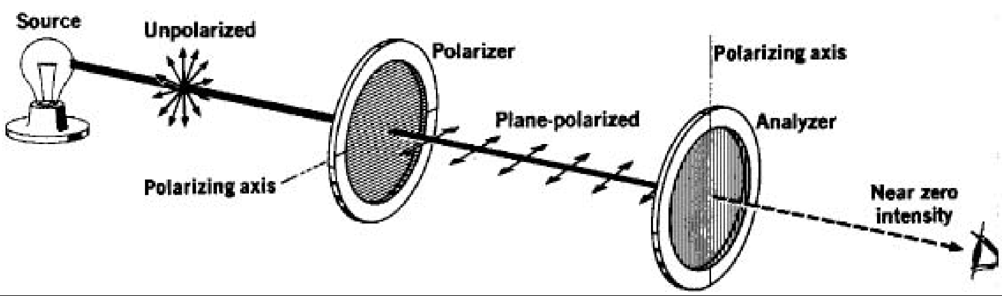
Horizontal polarization: Waves only vibrate in the horizontal plane.



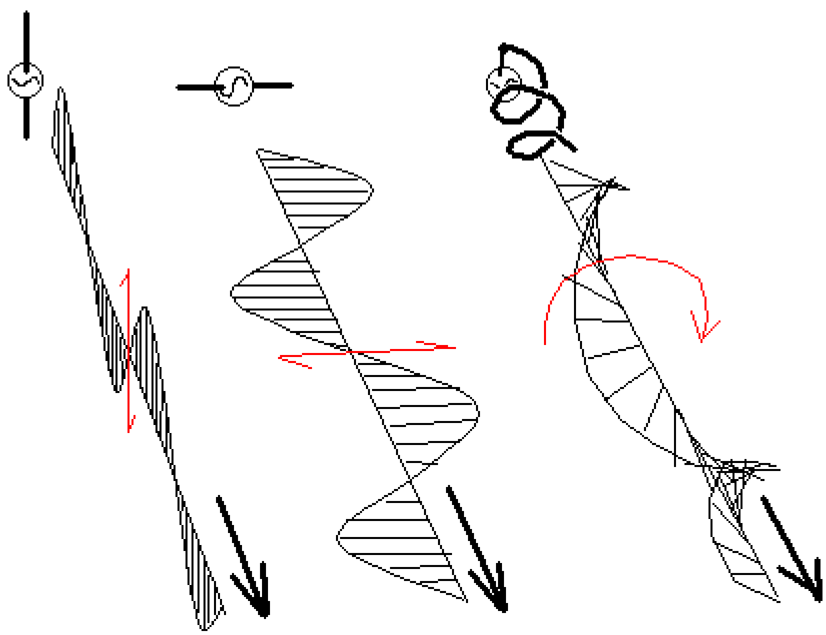
Polarization

Causing light to vibrate in one plane



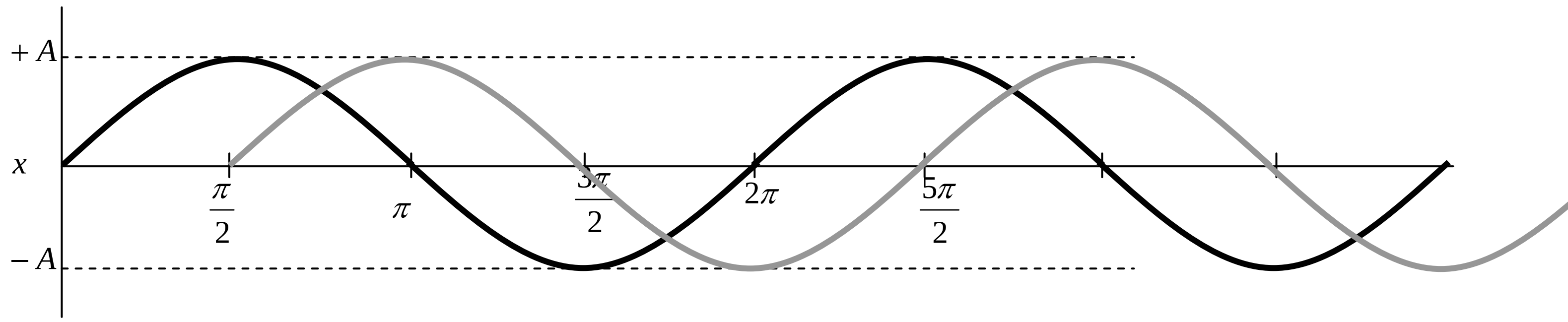


The waves below show three types of polarization
Vertical, Horizontal and Circular



Phase Shift

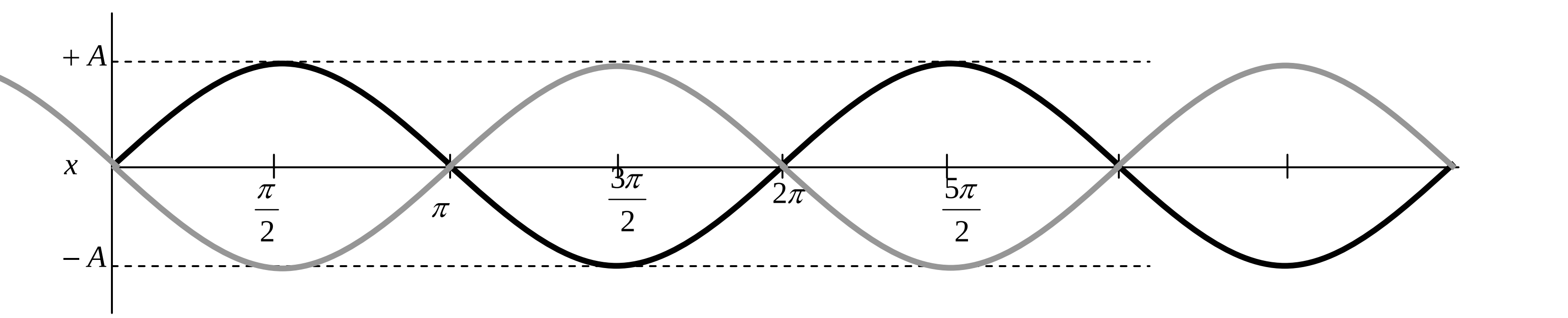
The concept of the phase shift is a fundamental component to wave theory. It is the *phase shift* that is responsible of the effects known as constructive and destructive interference.



These two waves are out of phase by $\frac{\pi}{2}$ where the second wave (grey) is trailing. Using the generic SHM equations

$$x = A\sin\theta$$

$$x = A\sin\left(\theta - \frac{\pi}{2}\right)$$



These two waves are out of phase by π where the second wave (grey) is leading. Using the generic SHM equations

$$x = A\sin\theta$$

$$x = A\sin(\theta + \pi)$$