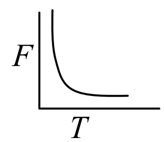
Previously it was demonstrated how the relationship between dependent and independent variables in any given set of data can be found through **graphing** or by analysing the **multipliers** within the set.

| F | T |
|---|---|
| 100 | 0.01 |
| 200 | 0.005 |
| 400 | 0.0025 |
| 500 | 0.002 |




**Issues:**

1. limited by the ability to graph the data
2. the reliability of the data
3. the complexity of the relationship

Some non-linear relationships can be very difficult to determine by using the above methods, hence the need for a second approach; **logarithms and exponents**

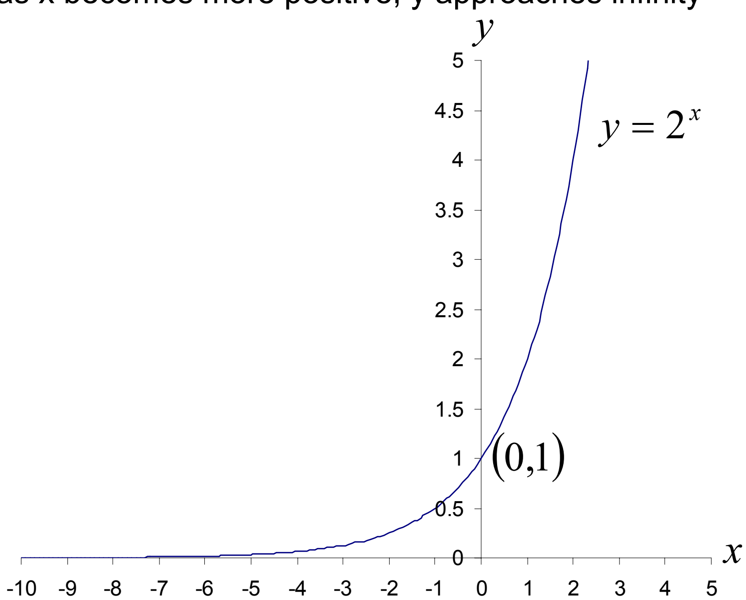
**The exponential function**

- functions that are in the following form

  $y = ma^x$ where $\{y, x, m, a \in R\}$ *and* $\{y \times m \geq 0\}$

- **example**: $y = 2^x$

  as x becomes more negative, y approaches 0
  as x becomes more positive, y approaches infinity

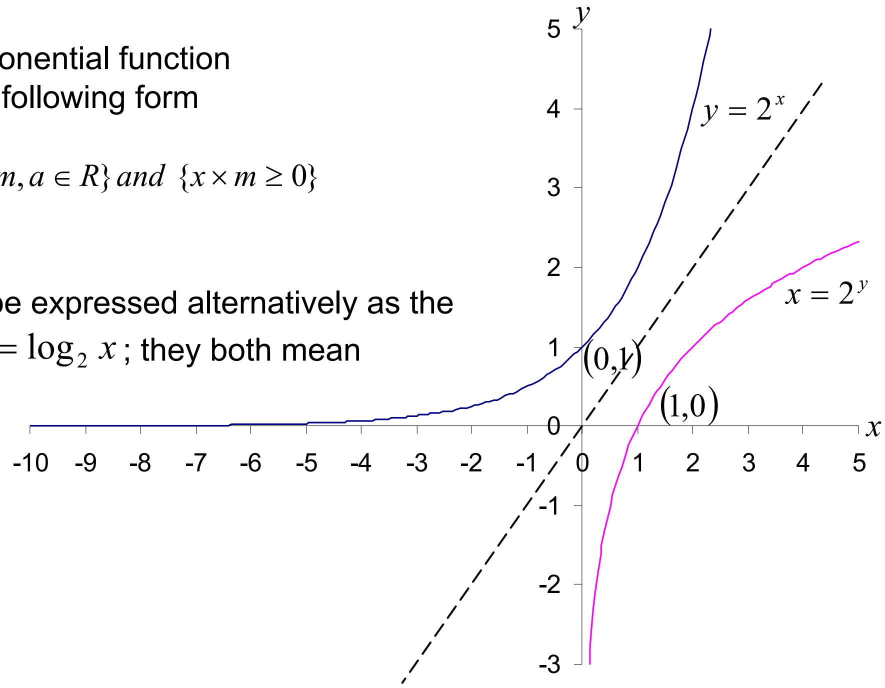


**The logarithmic Function**

- it is the inverse of the exponential function
- these functions are in the following form

$x = ma^y$ where $\{y, x, m, a \in R\}$ *and* $\{x \times m \geq 0\}$

- **Example** $x = 2^y$
- the function $x = 2^y$ can be expressed alternatively as the **logarithmic function**, $y = \log_2 x$; they both mean the exact same thing, the only difference is that the equation is expressed in terms of y as a function of x.




**Sample Questions**

a) find $\log_3$ of 9

answer:

$9 = 3^2$

$2 = \log_3 9$

b) $y = \log_4 16$

answer:

$16 = 4^y$

$4^2 = 4^y$

$2 = y$

$y = 2$

c) $2 = \log_3 x$

d) Find $\log_9$ of $\frac{1}{9}$

e) $y = \log_{10} 0.1$

f) $y = \log_5 5^{\sqrt{3}}$

g) $\log_8 4$

h) $\log_4 \sqrt{2} = y$

**Log Rules**

- $\log_a(nm) = \log_a(n) + \log_a(m)$
- $\log_a\left(\frac{n}{m}\right) = \log_a(n) - \log_a(m)$
- $\log_a(n^m) = m\log_a(n)$
- $\log_a(1) = 0$
- $\log_a(0) = undefined$
- $\log(x)$ is assumed to mean $\log_{10}(x)$; this is the most commonly used logarithm

**Q: How are logs useful in determining relationships?**

**A:** We know relationships can be direct, indirect, linear, non-linear, or exponential.

We have seen the following forms.

- $y \propto x$
- $y \propto x^2$
- $y \propto x^3$
- $y \propto \sqrt{x}$
- $y \propto \frac{1}{x}$
- $y \propto \frac{1}{x^2}$

These only address the **proportionality**; to actually find the relationship / formula, we need to look at the general form.

$$y = ax^n$$

This assumes our relationship passes through the origin. Examples: $y = 2x^2$, or $d = \frac{1}{2}at^2$

In the standard form ( $y = ax^n$ ), $y$ and $x$ are measured in the experiment, $a$ and $n$ need to be solved for.

To solve for a relationship like this

$$y = ax^n$$
$$\log(y) = \log(ax^n)$$
$$\log(y) = \log(a) + \log(x^n)$$
$$\log(y) = \log(a) + n\log(x)$$
$$\log(y) = n\log(x) + \log(a)$$

if you notice, the final equation is in the form y=mx+b, where m and b are unknown, to solve an equation like this you need **two equations** to solve for the **two unknowns**

**Sample problem**: let's find the relationship between $F$ and $T$ using the logarithm method.

| F | T |
|---|---|
| 100 | 0.01 |
| 200 | 0.005 |
| 400 | 0.0025 |
| 500 | 0.002 |

We assume that this data set passes through the origin, hence we are looking for something in the general form

$$y = ax^n$$

$$F = aT^n$$

$$\log(F) = \log(aT^n)$$

$$\log(F) = \log(a) + \log(T^n)$$

$$\log(F) = \log(a) + n\log(T)$$

$$\boxed{\log(F) = \log(a) + n\log(T)}$$

Equation 1: using (0.01, 100)

$$\log(F) = \log(a) + n\log(T)$$
$$\log(100) = \log(a) + n\log(0.01) \quad Eq(1)$$

Equation 2: using (0.005, 200)

$$\log(F) = \log(a) + n\log(T)$$
$$\log(200) = \log(a) + n\log(0.005) \quad Eq(2)$$

Eliminate (2) – (1)

$$\log(200) = \log(a) + n\log(0.005)$$
$$\underline{\log(100) = \log(a) + n\log(0.01)}$$
$$\log(200) - \log(100) = n\log(0.005) - n\log(0.01)$$
$$\log(200) - \log(100) = n(\log(0.005) - \log(0.01))$$
$$\log\left(\frac{200}{100}\right) = n\left(\log\frac{0.005}{0.01}\right)$$
$$\log(2) = n\log(0.5)$$
$$n = \frac{\log(2)}{\log(0.5)}$$
$$n = -1$$

Sub $n = -1$ into $F = aT^n$

$$F = aT^n$$
$$(100) = a(0.01)^{-1}$$
$$100 = a(100)$$
$$a = 1$$

$\therefore F = T^{-1}$ or $\boxed{F = \frac{1}{T}}$