Measurement and Error

A1: Measurement is the process of comparing or matching an unknown object with a known object.

A2: All measurement involves guessing or estimation which ultimately is the source of all errors.

A3: Any measured value has a numerical value which depends on the instrument used and how it is used. The choice of instrument also determines how large a guess needs to be made.

A4: In any mathematical operation involving the use of measurements, the answer shall not be more accurate than the values used to obtain the answer.

The above statements give rise to the following terms and procedures:

B1: Significant Digits — All digits that arise out of a measurement are termed as significant digits.

B2: Accuracy — The accuracy of any measurement is determined by the number of significant digits in the measurement.

B3: Precision — The precision of any measurement is determined by the number of decimal places in the measurement.

B4: Reliability — A measurement is said to be reliable if repeated measurements result in the same value.

B5: Percent Error (P.E.) — The comparison of error is made in the form of a percentage value. It is given by the formula

$$P.E. = \frac{m.v. - a.v.}{a.v.} \times 100$$

m.v. - measured value
a.v. - accepted value

B6: Arithmetic Mean ($\ddot{x}$) — The arithmetic mean represents the best value obtainable from a series of measurements. Given by the formula

$$\ddot{x} = \sum_{n=1}^{k} \frac{x_n}{k}$$

x_n - sum of all values

k - number of values

B7: Deviation (δx_n) The difference between a measurement and the arithmetic mean is called the deviation. It is defined by the formula $\delta x_n = |x_n - \bar{x}|$

B8: Average Deviation The average deviation is a measure of the accuracy of the experiment. It is defined as the sum of the deviations divided by the number of measurements.

B9: Standard Deviation (S.D.)

$$S.D. = \sqrt{\frac{\sum_{n-1}^{k} (\delta x_n)^2}{k-1}}$$

The standard deviation is a formula which involves an averaging method of the squares of the deviations of each measurement.

The value obtained from this formula means that 2/3 of all the measurements taken should lie within the region $\bar{x} \pm S.D.$ If it turns out that this is not the case, then probably not enough measurements have been taken to validate this statistic.

The following should be kept in mind whenever a measurement is taken:

C1: All scales on instruments should be read so that the last digit recorded in the measurement is a guess that is made between the smallest marked divisions.

C2: When converting from one unit to another, the number of significant digits does not change.

C3: The units that accompany a measurement must always be subjected to mathematical operations similar to those applied to the measurement.

C4: The following guidelines should be used in determining which digits of a measurement can be considered significant:

- All non zero digits are significant.
- Zeros in-between non zero digits are significant.
- Zeros at the beginning of any measurement are not significant.
- Zeros at the end of a measurement may or may not be significant. If the purpose of the zeros is to indicate magnitude, then the zeros are not significant.

Since all measurements contain a guess, whenever measurements are combined by means of mathematical operations, the answer also contains an error. In most cases the error in the answer is greater than the error in the individual measurements. To ensure that the answer is not expressed as being more accurate than the measurements that were used to obtain the answer, the following guidelines should be kept in mind:

D1: When adding or subtracting two measurements, the answer must have the same number of decimal places as the measurement with the least number of decimal places. This is accomplished by rounding off.

D2: When performing mathematical operations other than addition or subtraction, the answer must be rounded to the number of significant digits that is contained in the measurement that has fewest significant digits.

D3: Numbers that are exact counts, defined fractions or declared by definition are considered to be perfectly precise.

D4: There are three ways in which the error in any measurement can be reduced:

- By using a more precise instrument.
- By measuring a large quantity of whatever is being measured.
- By repeating the measurement many times. If repeated measurements are made of the same quantity, then the average (mean) of the values is used.