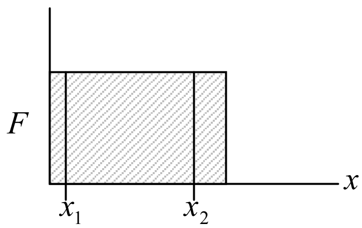
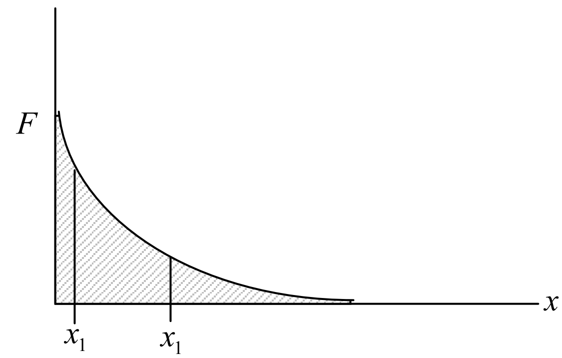
Work is defined as a **force** through a **displacement**. Graphically represented as the area under a F vs. x curve



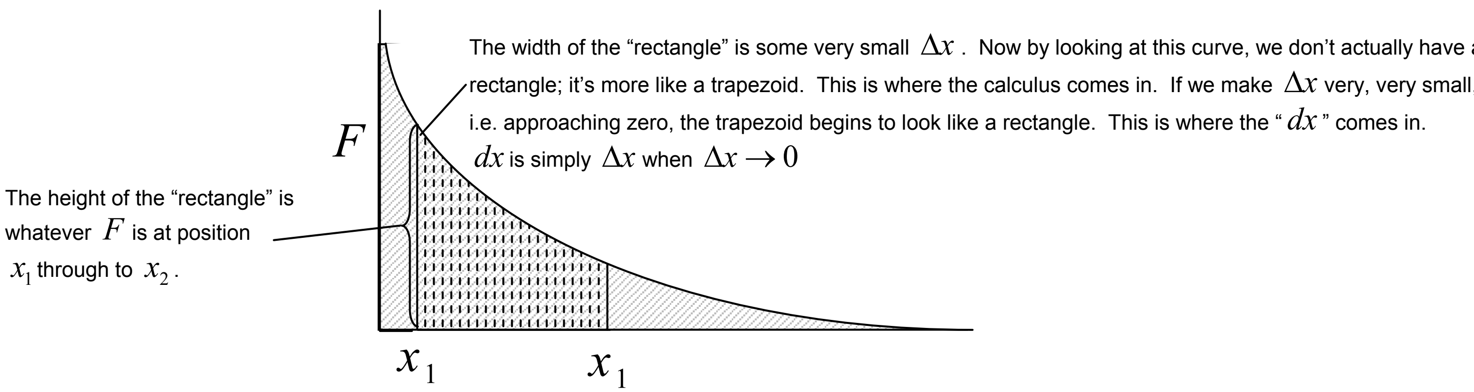
$$W = F \cdot x$$

$$\begin{aligned} W &= \int_{x_1}^{s_2} F\,dx \\ &= Fx \Big/_{x_1}^{x_2} \\ &= Fx_2 - Fx_1 \\ &= F\Delta x \end{aligned}$$

Finding the area beneath F vs. x curve is a trivial matter when F is constant, however, it becomes significantly more difficult when F is not linear. In these cases one must use integration (calculus) in order to determine the area beneath the curve. Although this is beyond the scope of the course, a partial derivation of this equation will be shown just for interest sake.



The integral is an interesting mathematical property. In reality it is simply the sum of the areas of several "rectangular slices"



Consider the force of gravity

The force of gravity is an attractive force. It always pulls an object in. Because of this, gravity is always considered to be negative. For example, if we launch a $1.0kg$ projectile up with an initial velocity of $5.0m/s$, we intuitively know that the velocity will eventual reach $0m/s$ at the maximum height. Therefore, the net force is negative. This implies that gravity will do negative work on the kinetic energy.

$$\begin{aligned} W &= \Delta E_k \\ &= E_{k2} - E_{k1} \\ &= \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2 \\ &= \frac{1}{2}(1.0)(0)^2 - \frac{1}{2}(1.0)(5.0)^2 \\ &= -12.5J \end{aligned}$$

This, of course, will translate into an increase in gravitation potential energy, hence preserving the concept of conservation of energy. The important thing to note is that the **negative** force does **negative** work on the object's **kinetic energy** but it translates into a **positive** increase in **gravitational potential energy**.

We know that the increase of gravitational potential energy comes at the expense of an object's initial kinetic energy. If we consider the **kinetic energy only,**

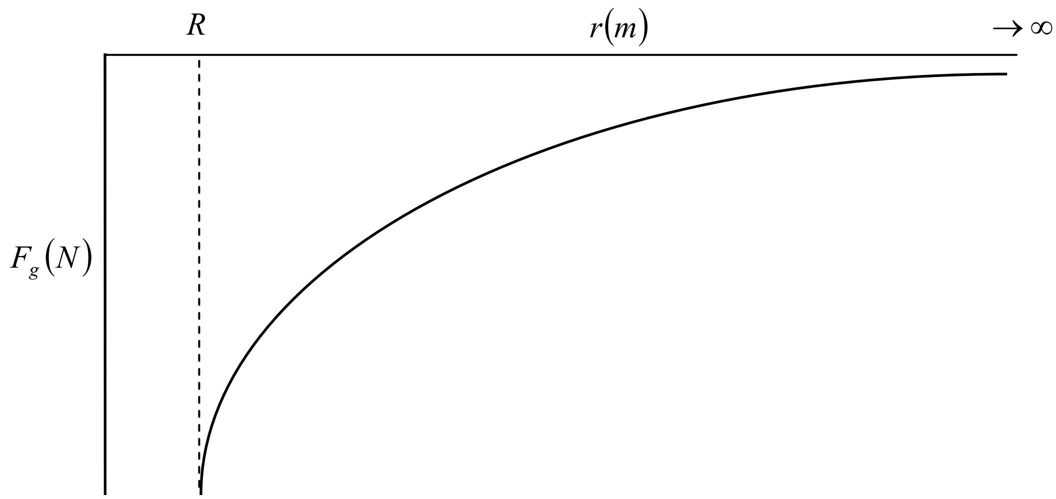
$$W = -F_g \Delta h$$

Note: if we were talking about the **increase** of **gravitational energy** we would have the formula

$$W = +F_g \Delta h$$

For this analysis we consider **ONLY** the **kinetic energy.**

The following graph represents the **force of gravity** compared to **the radial distance** from the center of some celestial body.



To find amount of work done on an object to move it from the surface of the celestial body to infinity, can be found using the following integral.

$$\begin{aligned}
W &= \int_R^\infty - F_g \, dr \\
&= \int_R^\infty \left(\frac{-GM_1M_2}{r^2} \right) dr \\
&= \frac{+GM_1M_2}{r} \Big/_R^\infty \\
&= +GM_1M_2 \left[\frac{1}{r} \right] \Big/_R^\infty \\
&= GM_1M_2 \left[\frac{1}{\infty} - \frac{1}{R} \right] \\
&= GM_1M_2 \left[0 - \frac{1}{R} \right] \\
&= GM_1M_2 \left[-\frac{1}{R} \right] \\
&= -\frac{GM_1M_2}{R} \\
E_g &= -\frac{GM_1M_2}{R}
\end{aligned}$$

This implies that the energy at $\infty = 0J$, and at any other point it is negative.