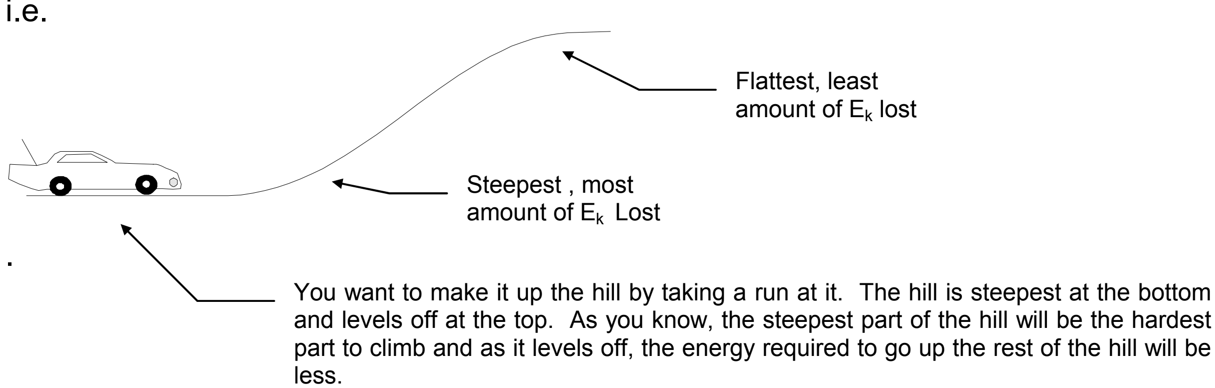
Section 10.6 Escape Velocity

As discussed earlier, there are two types of potential energies, positive potentials which tend to repel and negative potentials which tend to attract. These attractive potentials are often called **"potential wells"**. When stuck in a **"potential well"**, you are trapped. The only way to escape is to have enough energy to over come the negative potential.
i.e.

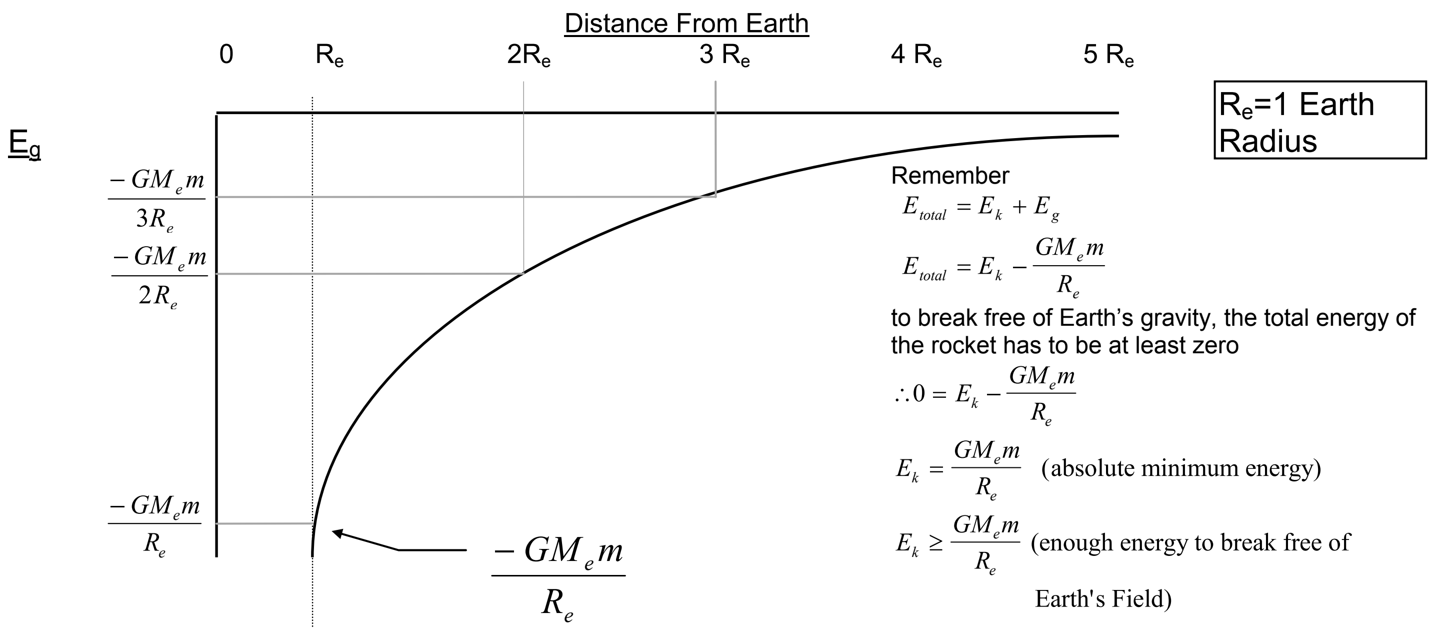


.

Same thing happens to objects stuck on earth. The hardest part of going into space is during the part of the launch closest to earth ($F_g = \frac{Gm_1m_2}{d^2}$). When d is smallest, F_g is greatest. Three situations are possible for a car "gunning it" up the hill.

1. The car's velocity (E_k) is not high enough, goes part way up and then rolls back down
2. The car has just enough velocity to make it to the top of the hill and comes to rest.
3. the car has more velocity than it need to over come the hill and keeps going.

If we look at the Earth's gravitational potential field, the strongest attractive potential (or negative potential) is located at the Earth's surface.



Remember

$$E_{total} = E_k + E_g$$

$$E_{total} = E_k - \frac{GM_em}{R_e}$$

to break free of Earth's gravity, the total energy of the rocket has to be at least zero

$$\therefore 0 = E_k - \frac{GM_em}{R_e}$$

$$E_k = \frac{GM_em}{R_e} \quad \text{(absolute minimum energy)}$$

$$E_k \geq \frac{GM_em}{R_e} \quad \text{(enough energy to break free of Earth's Field)}$$

Since we know that $E_k \geq \frac{GM_e m}{R_e}$ in order to escape Earth's gravitational pull, we can easily find the necessary escape velocity

$$\frac{1}{2}mv^2 \geq \frac{GM_e m}{R_e}$$

$$\frac{1}{2}v^2 \geq \frac{GM_e}{R_e}$$

$$v \geq \sqrt{\frac{2GM_e}{R_e}}$$

$$v \geq \sqrt{\frac{2(6.67\times10^{-11})(5.98\times10^{24}}{6.4\times10^6}}$$

$$v \geq 1.12\times10^4 \ m/s$$

Any velocity less than this will cause the rocket to fall back to earth. Orbiting satellites are in a constant state of "free fall". They just keep falling past the earth.

Satellites bound to Earth

How much energy is required to put a satellite in a stable orbit? This is surprisingly easy to figure out. From what we know already.

$$F_c = F_g$$

$$\frac{mv^2}{R} = \frac{GM_e m}{R^2}$$

$$mv^2 = \frac{GM_e m}{R}$$

$$\frac{mv^2}{2} = \frac{GM_e m}{2R}$$

Borrowing from the fact that at any Orbiting radius R, the centripetal force must equal the gravitational force But the total energy in the system is constant. Using some very simple algebra, we can turn this force equation into an expression of energy.

$$E_{total} = E_k + E_g$$

$$= \frac{1}{2}mv^2 - \frac{GM_e m}{R}$$

$$= \frac{GM_e m}{2R} - \frac{GM_e m}{R}$$

$$= -\frac{GM_e m}{2R}$$

$$= \frac{1}{2}E_g$$

To maintain a stable orbit, you only need to provide enough kinetic energy to over come ½ of the Earth's potential energy at that distance R from the center of the earth. If you want to break completely, you have to provide an additional ½ E_g. This remaining energy barrier is called the binding energy.