$$v_E = \sqrt{\frac{GM_S}{r_E}}$$

$$= \sqrt{\frac{(6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2)(1.99 \times 10^{30}\ \text{kg})}{1.49 \times 10^{11}\ \text{m}}}$$

$$v_E = 2.98 \times 10^4\ \text{m/s}$$

Venus travels at a speed of 3.51×10^4 m/s and Earth travels more slowly at a speed of 2.98×10^4 m/s around the Sun.

Practice

Understanding Concepts

Refer to Appendix C for required data.

1. Why does the Moon, which is attracted by gravity toward Earth, not fall into Earth?
2. Why does the gravitational force on a space probe in a circular orbit around a planet not change the speed of the probe?
3. A satellite is in circular orbit 525 km above the surface of Earth. Determine the satellite's (a) speed and (b) period of revolution.
4. A satellite can travel in a circular orbit very close to the Moon's surface because there is no air resistance. Determine the speed of such a satellite, assuming the orbital radius is equal to the Moon's radius.

Applying Inquiry Skills

5. (a) Write a proportionality statement indicating the relationship between the speed of a natural or artificial satellite around a central body and the radius of the satellite's orbit.
   (b) Sketch a graph of that relationship.

Making Connections

6. Space junk is becoming a greater problem as more human-made objects are abandoned in their orbits around Earth. Research this problem using the Internet or other appropriate publications, and write a brief summary of what you discover.

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Answers

3. (a) 7.60×10^3 m/s
   (b) 5.71×10^3 s (or 1.59 h)
4. 1.68×10^3 m/s

Kepler's Laws of Planetary Motion

Centuries before telescopes were invented, astronomers made detailed observations of the night sky and discovered impressive and detailed mathematical relationships. Prior to the seventeenth century, scientists continued to believe that Earth was at or very near the centre of the universe, with the Sun and the other known planets (Mercury, Venus, Mars, Jupiter, and Saturn) travelling in orbits around Earth. Using Earth as the frame of reference, the "geocentric model" of the universe was explained by introducing complicated motions (**Figure 2**).

The detailed observations and analysis needed to invent these complex orbits were amazingly accurate and allowed scientists to predict such celestial events as solar and lunar eclipses. However, the causes of the motions were poorly understood. Then in 1543, Polish astronomer Nicolas Copernicus (1473–1543) published a book in which he proposed the "heliocentric model" of the solar system in which the planets revolve around the Sun. He deduced that the planets closer to the Sun have a higher speed than those

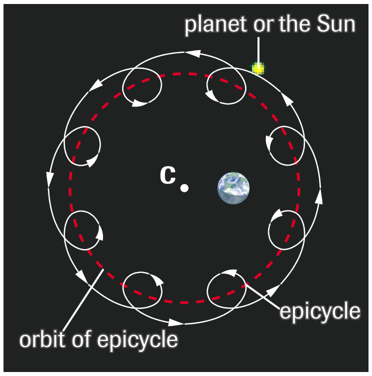

Figure 2
Using Earth as the frame of reference, the motion of the Sun and the other planets is an orbit called an *epicycle*, which itself is in an orbit around point C, located away from Earth.

We have proven that the constant for the Sun depends only on the mass of the Sun. This relationship, however, applies to any central body about which other bodies orbit. For example, the constant for Earth C_E depends on the mass of Earth M_E and applies to the Moon or to any artificial satellite in orbit around Earth:

$$C_E = \frac{GM_E}{4\pi^2} = \frac{r_{Moon}{}^3}{T_{Moon}{}^2}$$

Today's astronomers use sophisticated Earth-bound and orbiting telescopes to gather accurate data of the motions of celestial bodies, as well as advanced computing and simulation programs to analyze the data. But astronomers will always admire the accuracy and unending hard work of the Renaissance astronomers, especially Tycho and Kepler.

LEARNING TIP

More about Kepler's Third-Law Constant

The constant of proportionality C is defined in this text as the ratio of r^3 to T^2, which is equal to the ratio $\frac{GM}{4\pi^2}$ and is measured in metres cubed per second squared. The constant could also be written as the ratio of T^2 to r^3, or $\frac{4\pi^2}{GM}$ and is measured in seconds squared per metre cubed. This latter case is found in some references.

Practice

Understanding Concepts

7. If the solar system were considered to be an isolated system, which model (geocentric or heliocentric) is the noninertial frame of reference? Explain your answer.

8. Why did Tycho not gather any data from the planets beyond Saturn?

9. Between March 21 and September 21, there are three days more than between September 21 and March 21. These two dates are the spring and fall equinoxes when the days and nights are of equal length. Between the equinoxes, Earth moves 180° around its orbit with respect to the Sun. Using Kepler's laws, explain how you can determine the part of the year during which the Earth is closer to the Sun.

10. Using the planetary data in Appendix C, calculate the ratio $\frac{r^3}{T^2}$ for each planet, and verify Kepler's third law by confirming that $r^3 \propto T^2$.

11. (a) What is the average value (in SI base units) of the constant of proportionality in $r^3 \propto T^2$ that you found in question 10?
(b) Use your answer in (a) to determine the mass of the Sun.

12. (a) Use the data of the Moon's motion (refer to Appendix C) to determine Kepler's third-law constant C_E to three significant digits for objects orbiting Earth.
(b) If a satellite is to have a circular orbit about Earth ($m_E = 5.98 \times 10^{24}$ kg) with a period of 4.0 h, how far, in kilometres, above the centre of Earth must it be? What must be its speed?

Applying Inquiry Skills

13. Go back to the ellipses you drew in the Try This Activity at the beginning of Chapter 6 and label one focus on each ellipse the "Sun." As accurately as possible, draw diagrams to illustrate Kepler's second law of planetary motion. Verify that a planet travels faster when it is closer to the Sun. (Your diagram for each ellipse will resemble **Figure 6**; you can use approximate distances along the arcs to compare the speeds.)

Making Connections

14. Astronomers have announced newly discovered solar systems far beyond our solar system. To determine the mass of a distant star, they analyze the motion of a planet around that star.
(a) Derive an equation for the mass of a central body, around which another body revolves in an orbit of known period and average radius.
(b) If a planet in a distant solar system cannot be observed directly, its effect on the central star might be observed and used to determine the radius of the planet's orbit. Describe how this is possible for a "main-sequence star" whose mass can be estimated by its luminosity. (Assume there is a single large-mass planet in orbit around the star and that the star has an observable wobble.)

Answers

11. (a) 3.36×10^{18} m^3/s^2
(b) 1.99×10^{30} kg

12. (a) 1.02×10^{13} m^3/s^2
(b) 1.3×10^4 km; 5.6×10^3 m/s

14. (a) $M = \frac{4\pi^2 r^3}{GT^2}$

SUMMARY Orbits and Kepler's Laws

- The orbits of planets are most easily approximated as circles even though they are ellipses.
- Kepler's first law of planetary motion states that each planet moves around the Sun in an orbit that is an ellipse, with the Sun at one focus of the ellipse.
- Kepler's second law of planetary motion states that the straight line joining a planet and the Sun sweeps out equal areas in space in equal intervals of time.
- Kepler's third law of planetary motion states that the cube of the average radius r of a planet's orbit is directly proportional to the square of the period T of the planet's orbit.

Section 6.2 Questions

Understanding Concepts

Refer to Appendix C for required data.

1. Apply one of Kepler's laws to explain why we are able to observe comets close to Earth for only small time intervals compared to their orbital periods. (*Hint:* A comet's elliptical orbit is very elongated.)
2. Earth is closest to the Sun about January 4 and farthest from the Sun about July 5. Use Kepler's second law to determine on which of these dates Earth is travelling most rapidly and least rapidly.
3. A nonrotating frame of reference placed at the centre of the Sun is very nearly an inertial frame of reference. Why is it not exactly an inertial frame of reference?
4. An asteroid has a mean radius of orbit around the Sun of 4.8×10^{11} m. What is its orbital period?
5. If a small planet were discovered with an orbital period twice that of Earth, how many times farther from the Sun is this planet located?
6. A spy satellite is located one Earth radius above Earth's surface. What is its period of revolution, in hours?
7. Mars has two moons, Phobos and Deimos (Greek for "Fear" and "Panic," companions of Mars, the god of war). Deimos has a period of 30 h 18 min and an average distance from the centre of Mars of 2.3×10^4 km. The period of Phobos is 7 h 39 min. What is the average distance of Phobos from the centre of Mars?

Applying Inquiry Skills

8. Show that the SI base units of $\sqrt{\frac{GM}{r}}$ are metres per second.
9. Sketch the shape of a graph of r^3 as a function of T^2 for planets orbiting the Sun. What does the slope of the line on the graph indicate?

Making Connections

10. Galileo was the first person to see any of Jupiter's moons.
    (a) Relate this important event to the works of Tycho and Kepler by researching when Galileo first discovered that Jupiter had moons and how this discovery came to pass.
    (b) After discovering these moons, what would Galileo need to know to calculate Jupiter's mass?
    (c) Would Galileo have been able to determine Jupiter's mass when he first saw the moons, or would that calculation have had to wait for awhile? (*Hint:* Kepler's first two laws were published in 1609.)