Practice

Understanding Concepts

1. Determine the gravitational potential energy of the Earth–Moon system, given that the average distance between their centres is 3.84×10^5 km, and the mass of the Moon is 0.0123 times the mass of Earth.

2. (a) Calculate the change in gravitational potential energy for a 1.0-kg mass lifted 1.0×10^2 km above the surface of Earth.

(b) What percentage error would have been made in using the equation $\Delta E_g = mg\Delta y$ and taking the value of g at Earth's surface?

(c) What does this tell you about the need for the more exact treatment in most normal Earth-bound problems?

3. With what initial speed must an object be projected vertically upward from the surface of Earth to rise to a maximum height equal to Earth's radius? (Neglect air resistance.) Apply energy conservation.

Answers

1. -7.64×10^{28} J
2. (a) 1.0×10^6 J
   (b) 2%
3. 7.91×10^3 m/s

Answers

4. (a) 1.8×10^{32} J

   (b) perihelion; 1.8×10^{32} J

5. (a) -1.56×10^{10} J; -1.04×10^{10} J

   (b) 5.2×10^{9} J

   (c) 5.2×10^{9} J

LEARNING TIP

"Apo" and "Peri"

The prefix "apo" means away from and "geo" represents Earth, so apogee refers to the point in a satellite's elliptical orbit farthest from Earth. Furthermore, since "helios" represents the Sun, aphelion refers to the point in a planet's elliptical orbit farthest from the Sun. The prefix "peri" means around, so perihelion refers to the point in a planet's orbit closest to the Sun. What does perigee mean?

4. The distance from the Sun to Earth varies from 1.47×10^{11} m at perihelion (closest approach) to 1.52×10^{11} m at aphelion (farthest distance away).

(a) What is the maximum change in the gravitational potential energy of Earth during one orbit of the Sun?

(b) At what point in its orbit is Earth moving the fastest? What is its maximum change in kinetic energy during one orbit? (Think about energy conservation.)

Making Connections

5. A satellite of mass 5.00×10^{2} kg is in a circular orbit of radius $2r_E$ around Earth. Then it is moved to a circular orbit of radius $3r_E$.

(a) Determine the satellite's gravitational potential energy in each orbit.

(b) Determine the change in gravitational potential energy from the first orbit to the second orbit.

(c) Determine the work done in moving the satellite from the first orbit to the second orbit. Apply energy conservation.

Escape from a Gravitational Field

We have seen that any two masses have a gravitational potential energy of $E_g = -\frac{GMm}{r}$ at a separation distance r. The negative value of this potential energy is characteristic of a *potential well*, a name derived from the shape of the graph of the gravitational potential energy as a function of separation distance (**Figure 4**).

For example, a rocket at rest on Earth's surface has the value of E_g, given by point A on the graph in **Figure 4**. Since the kinetic energy E_K of the rocket is zero, its total energy E_T would also be represented by point A, and the rocket would not leave the ground. However, suppose the rocket is launched at a speed such that its kinetic energy is represented by the distance AB on the graph. Now its total energy $E_T = E_g + E_K$ is represented by point B, and the rocket begins to rise. As its altitude increases, E_g increases

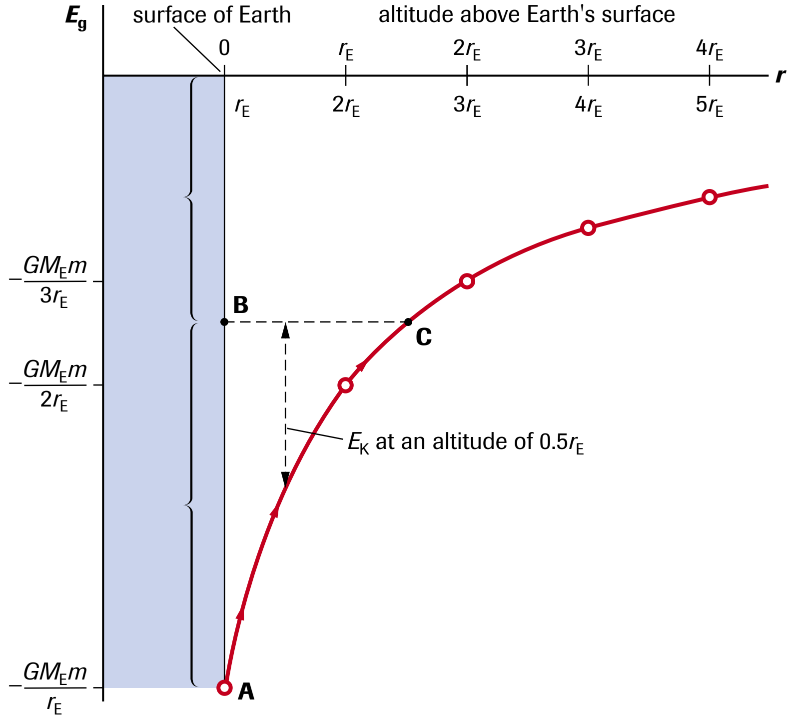
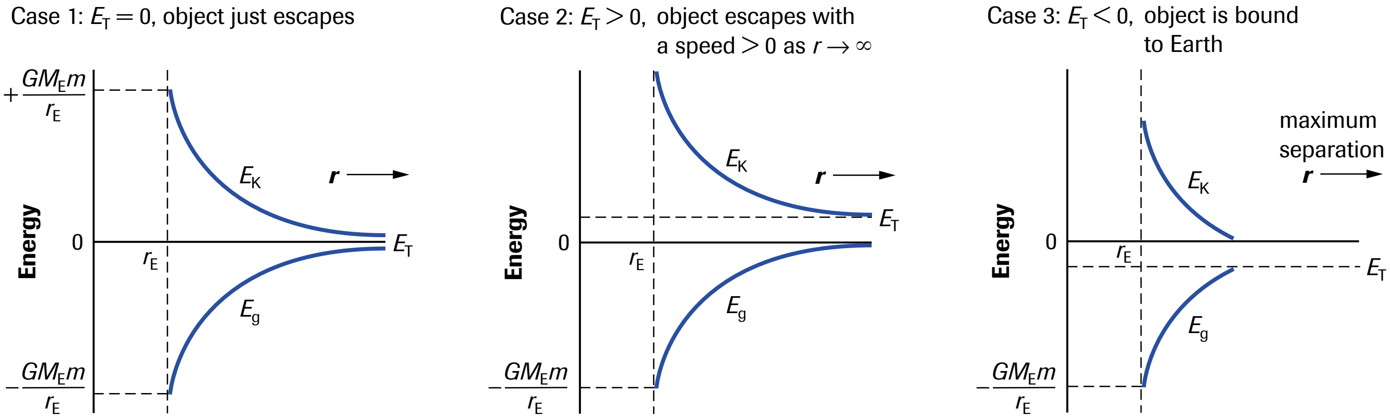


Figure 4
This graph of the gravitational potential energy as a function of the altitude above Earth's surface illustrates Earth's potential well.



Case 1: $E_T = 0$, object just escapes
$+\frac{GM_E m}{r_E}$
E_K
r
Energy
0
r_E
E_T
E_g
$-\frac{GM_E m}{r_E}$
Case 2: $E_T > 0$, object escapes with a speed > 0 as $r \to \infty$
E_K
r
E_T
Energy
0
r_E
E_g
$-\frac{GM_E m}{r_E}$
Case 3: $E_T < 0$, object is bound to Earth
maximum separation
r
E_K
Energy
0
r_E
E_T
E_g
$-\frac{GM_E m}{r_E}$

As an example, assume that a certain black hole results from the collapse of a star that has a mass 28 times the Sun's mass. Since the minimum escape speed is $v_e = c$, we have

$$\frac{mv_e^2}{2} = \frac{GMm}{r}$$

$$v_e^2 = \frac{2GM}{r}$$

$$r = \frac{2GM}{v_e^2}$$

$$= \frac{2GM}{c^2}$$

$$= \frac{2(6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2)(28 \times 1.99 \times 10^{30}\ \text{kg})}{(3.00 \times 10^8\ \text{m/s})^2}$$

$$= 8.26 \times 10^4\ \text{m}$$

$$r = 82.6\ \text{km}$$

Since light cannot escape from a black hole, the only way a black hole can be detected is indirectly. Material that is close enough to the black hole gets sucked in, and as it does so, the material emits X rays that can be detected and analyzed.

The celestial mechanics analyzed in this chapter is not a complete picture. You will learn more about high-speed and high-energy particles when you study Einstein's special theory of relativity in Chapter 11.

DID YOU KNOW?

First Black Hole Discovery

In 1972, Professor Tom Bolton, while working at the University of Toronto's David Dunlap Observatory in Richmond Hill, Ontario, was investigating a point in space, Cygnus X-1, because it was a source of X rays. It turned out to be one of the most significant discoveries in astronomy: a black hole. This was the first evidence to support the existence of black holes, which were previously hypothetical objecs.

Practice

Understanding Concepts

6. Does the escape speed of a space probe depend on its mass? Why or why not?

7. Jupiter's mass is 318 times that of Earth, and its radius is 10.9 times that of Earth. Determine the ratio of the escape speed from Jupiter to the escape speed from Earth.

8. The Moon is a satellite of mass 7.35×10^{22} kg, with an average distance of 3.84×10^8 m from the centre of Earth.
- (a) What is the gravitational potential energy of the Moon–Earth system?
- (b) What is the Moon's kinetic energy and speed in circular orbit?
- (c) What is the Moon's binding energy to Earth?

9. What is the total energy needed to place a 2.0×10^3-kg satellite into circular Earth orbit at an altitude of 5.0×10^2 km?

10. How much additional energy would have to be supplied to the satellite in question 9 once it was in orbit, to allow it to escape from Earth's gravitational field?

11. Consider a geosynchronous satellite with an orbital period of 24 h.
- (a) What is the satellite's speed in orbit?
- (b) What speed must the satellite reach during launch to attain the geosynchronous orbit? (Assume all fuel is burned in a short period. Neglect air resistance.)

12. Determine the Schwartzchild radius, in kilometres, of a black hole of mass 4.00 times the Sun's mass.

Applying Inquiry Skills

13. Sketch the general shape of the potential wells of both Earth and the Moon on a single graph. Label the axes and use colour coding to distinguish the line for Earth from the line for the Moon.

Making Connections

14. (a) Calculate the binding energy of a 65.0-kg person on Earth's surface.
- (b) How much kinetic energy would this person require to just escape from the gravitational field of Earth?
- (c) How much work is required to raise this person by 1.00 m at Earth's surface?
- (d) Explain why one of NASA's objectives in designing launches into space is to minimize the mass of the payload (including the astronauts).

Answers

7. 5.40:1

8. (a) -7.63×10^{28} J
   (b) 3.82×10^{28} J; 1.02×10^3 m/s
   (c) 3.82×10^{28} J

9. 6.7×10^{10} J

10. 5.80×10^{10} J

11. (a) 3.1×10^3 m/s
    (b) 1.1×10^4 m/s

12. 11.8 km

14. (a) 4.06×10^9 J
    (b) 4.06×10^9 J
    (c) 6.37×10^2 J

SUMMARY Gravitational Potential Energy in General

- The gravitational potential energy of a system of two (spherical) masses is directly proportional to the product of their masses, and inversely proportional to the distance between their centres.
- A gravitational potential energy of zero is assigned to an isolated system of two masses that are so far apart (i.e., their separation is approaching infinity) that the force of gravity between them has dropped to zero.
- The change in gravitational potential energy very close to Earth's surface is a special case of gravitational potential energy in general.
- Escape speed is the minimum speed needed to project a mass m from the surface of mass M to just escape the gravitational force of M.
- Escape energy is the minimum kinetic energy needed to project a mass m from the surface of mass M to just escape the gravitational force of M.
- Binding energy is the amount of additional kinetic energy needed by a mass m to just escape from a mass M.

Section 6.3 Questions

Understanding Concepts

1. How does the escape energy of a 1500-kg rocket compare to that of a 500-kg rocket, both initially at rest on Earth?

2. Do you agree or disagree with the statement, "No satellite can orbit Earth in less than about 80 min"? Give reasons. (*Hint:* The greater the altitude of an Earth satellite, the longer it takes to complete one orbit.)

3. A space shuttle ejects a 1.2×10^3-kg booster tank so that the tank is momentarily at rest, relative to Earth, at an altitude of 2.0×10^3 km. Neglect atmospheric effects.
(a) How much work is done on the booster tank by the force of gravity in returning it to Earth's surface?
(b) Determine the impact speed of the booster tank.

4. A space vehicle, launched as a lunar probe, arrives above most of Earth's atmosphere. At this point, its kinetic energy is 5.0×10^9 J and its gravitational potential energy is -6.4×10^9 J. What is its binding energy?

5. An artificial Earth satellite, of mass 2.00×10^3 kg, has an elliptical orbit with an average altitude of 4.00×10^2 km.
(a) What is its average gravitational potential energy while in orbit?
(b) What is its average kinetic energy while in orbit?
(c) What is its total energy while in orbit?
(d) If its perigee (closest position) is 2.80×10^2 km, what is its speed at perigee?

6. A 5.00×10^2-kg satellite is in circular orbit 2.00×10^2 km above Earth's surface. Calculate
(a) the gravitational potential energy of the satellite
(b) the kinetic energy of the satellite
(c) the binding energy of the satellite
(d) the percentage increase in launching energy required for the satellite to escape from Earth

7. (a) Calculate the escape speed from the surface of the Sun: mass $= 1.99 \times 10^{30}$ kg, radius $= 6.96 \times 10^8$ m.
(b) What speed would an object leaving Earth need to escape from our solar system?

8. The mass of the Moon is 7.35×10^{22} kg, and its radius is 1.74×10^6 m. With what speed must an object be projected from the its surface to reach an altitude equal to its radius?

9. A black hole has a Schwartzschild radius of 15.4 km. What is the mass of the black hole in terms of the Sun's mass?

Applying Inquiry Skills

10. Mars is a planet that could be visited by humans in the future.
(a) Generate a graph of Mars' potential well (using data from Appendix C) for a spacecraft of mass 2.0×10^3 kg that is launched from Mars. Draw the graph up to $5r_M$.
(b) On your graph, draw
(i) the line for the kinetic energy needed for the craft to just escape from Mars
(ii) the line of the total energy from Mars' surface to $5r_M$

11. (a) What is the theoretical Schwartzschild radius of a black hole whose mass is equal to the mass of Earth. Express your answer in millimetres.
(b) What does your answer imply about the density of a black hole?

Making Connections

12. How would the amount of fuel required to send a spacecraft from Earth to the Moon compare with the amount needed to send the same spacecraft from the Moon back to Earth? Explain. (Numerical values are not required.)