Planetary Mechanics
(a division of Centripetal Acceleration)

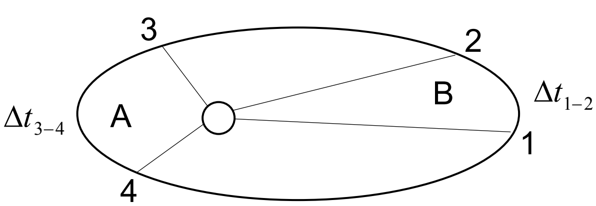
The ancient Greeks worshipped the stars and planets. At the time, the motions of the various planets were believed to be linked with the movements of the gods. They also believed that the earth was the center of the universe. This belief was challenged by **Copernicus** (1473-1543). The problem of retrograde motion of the planets could not be explained under a geocentric view of the universe but could be explained under a heliocentric view. The church didn't like the idea of a heliocentric universe because it conflicted with many of the churches beliefs. **Galileo** (1564-1642) was put on trial for supporting the **Copernican theory**. Part of the reason was that in one of his books, **Galileo** made reference to a "simple minded" supporter of the geocentric model, which the pope at the time was convinced by **Galileo's** enemies to be in reference to the Pontiff himself. The pope was not impressed by Galileo's arrogance and commanded Galileo to appear in front of the inquisition. The reputation of the inquisition's brutal and torturous methods of investigation was enough to make **Galileo** renounced his theories. It was not until after death when **Galileo's** theories resurfaced and were accepted. As a point of interest it should be noted that in 1982 a papal commission acknowledge that the church was wrong, it was not until 1992 when the church reversed the condemnation of **Galileo**. The theory was published in 1613 for a difference of 379 years. It was this level of intellectual oppression that earned this particular era the appropriate name, the Dark Ages.

Tycho Brahe made fantastic observations of the planets' motion. He was also responsible for perfecting the calendar. **Johann Kepler** continued work and discovered that the motion of the planets were elliptical not circular. By using **Brahe's** observations he came to these conclusions.

Kepler's Laws

1) The planets move about the sun in elliptical orbits with the sun at one of the foci.

2) The straight line joining the sun and the planets sweeps out equal areas in equal times.



$\Delta t_{1-2} = \Delta t_{3-4}$

and

$area\ A = area\ B$

3) The square of the period of revolution of a planet about the sun is proportional to the cube of its mean distance from the sun (true for every planet orbiting the sun)

$\frac{R^3}{T^2} = K$, K is the same for earth as it is for Jupiter, Mars, Etc.. (see Handout)

The earth is about 1.4x10[11]m from the sun and takes 3.16x10[7]s for one complete orbit. How long is Mars' year if it is 2.28x10[11]m from the sun?

Earth

$$K = \frac{R^3}{T^2} = \frac{(1.49\times10^{11})^3}{(3.16\times10^7)^2}$$

Mars

$$K = \frac{R^3}{T^2} = \frac{(2.28\times10^{11})^3}{T^2}$$

Since K is constant for both Mars and Earth.

$$\frac{(1.49\times10^{11})^3}{(3.16\times10^7)^2} = \frac{(2.28\times10^{11})^3}{T^2}$$

$$T^2 = 3.578\times10^{15}$$

$$= 5.98\times10^7 s$$

$$= 1.9\,years$$

Newton's laws of planetary mechanics

Newton took Kepler's work and ran. He discovered that all objects are innately attracted to each other by a force called gravity. He also discovered that the strength of this gravitational field is inversely proportional to the square of the distance between the objects.

$$F_g \propto \frac{1}{R^2}$$

How did such a bold conclusion come about? Through experimentation; we however have the convenience of space-age technology that enables us to observe this relationship by simply observing the motion of our own moon.

Given

$R_{E-M} = 3.8\times10^8 m$

$T = 2.36\times10^6 s$

Formula

$$a_c = \frac{4\pi^2 R}{T^2}$$

Centripetal acceleration

Solution

$$a_c = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2 3.8\times10^8}{(2.36\times10^6)^2} = 2.7\times10^{-3} m/s^2$$

We know at the earth's surface, the gravity is 9.8m/s^2 but the moon's orbit only exhibits a centripetal acceleration of 2.7x10^{-3}m/s^2.

It's should be evident that the centripetal motion of the moon is controlled by the Earth's force of gravity. We also now know that moon is 60R$_e$ (60 earth radii) away from earth. We can apply ***proportional reasoning*** to Newton's relationship $F_g \propto 1/R^2$

$$if\ F_g \propto \frac{1}{R^2} \quad and\ F_g = mg \quad \therefore g \propto \frac{1}{R^2}$$

$$g_E \propto \frac{1}{(R_E)^2}$$

$$g_M \propto \frac{1}{(R_M)^2}\ but\ R_M = 60R_E$$

$$g_M \propto \frac{1}{(60R_E)^2}$$

$$g_M \propto \frac{1}{60^2}\frac{1}{(R_E)^2}$$

$$g_M \propto \frac{1}{60^2} g_E$$

$$g_M \propto \frac{1}{3600}(9.8)$$

$$g_M \cong 2.7\times10^{-3} m/s$$

Conclusions, since $a \propto \frac{1}{R^2}$ and $F_g = ma_g$, therefore $F_g \propto \frac{1}{R^2}$ where F_g is the force of gravity between the centers of the two objects.

Newton's Law of Universal Gravitation

Centripetal acceleration of an orbiting object can be found using this formula $a_c = \frac{4\pi^2 R}{T^2}$,

Therefore Centripetal force required to keep an object in this orbit is $F_c = \frac{4\pi^2 mR}{T^2}$ ①

From Kepler's 3rd law, $K = \frac{R^3}{T^2}$ or $T^2 = \frac{R^3}{K}$ ②

Sub ② into ①

$$F_c = \frac{4\pi^2 mR}{R^3/K} = \frac{4\pi^2 mRK}{R^3} = \frac{4\pi^2 Km}{R^2}$$

or $\boxed{F_c = \left(4\pi^2 K\right)\frac{m}{R^2}}$ where m is the mass of the orbiting satellite and K is the Kepler Constant for the system

However, K was calculated for objects orbiting the sun. K would be different for an object orbiting earth because of a mass difference. We can get around this problem

For Objects Orbiting the Sun

$K_s \propto M_s$

$\therefore 4\pi^2 K_s \propto M_s$

$\therefore 4\pi^2 K_s = GM_s$

Where K_s is the Kepler's constant for objects orbiting the sun, M_s is the mass of the sun and G is a proportional constant

Sub $4\pi^2 K_s = GM_s$ back into

$F_c = \left(4\pi^2 K_s\right)\frac{m}{R^2}$ we get

$\boxed{F_c = GM_s \frac{m}{R^2} = \frac{GM_s m}{R^2}}$

and

For Objects Orbiting the Earth

$K_E \propto M_E$

$\therefore 4\pi^2 K_E \propto M_E$

$\therefore 4\pi^2 K_E = GM_E$

Where K_E is the Kepler's constant for objects orbiting the earth, M_E is the mass of the earth and G is a proportional constant.

Sub $4\pi^2 K_E = GM_E$ back into

$F_c = \left(4\pi^2 K_E\right)\frac{m}{R^2}$ we get

$\boxed{F_c = GM_E \frac{m}{R^2} = \frac{GM_E m}{R^2}}$

In General

$$\boxed{F_g = \frac{Gm_1 m_2}{d^2}}$$

Where F_g is the gravitational force, in Newtons (N) between two orbiting masses.

G is the gravitational constant of the universe where $G = 6.67 \times 10^{-11} N \cdot m^2 / kg^2$,

m_1 and m_2 are the masses, in kg, of the objects in orbiting system, and

d is the distance separating the center of the two masses.