At the beginning of the 20th century, scientists believed that after James Maxwell had bridged the concepts of electricity and magnetism together, all but a few minor problems in physics had been solved. Unfortunately, the honeymoon of physics only lasted a mere 30 years before Wilhem Roentgen discovered the existence of X-rays, which could not be explained by any current law of physics. X-rays were produced within a cathode ray tube by the collision of the electrons with the glass wall. After Roentgen's discovery, every researcher and his/her assistant jumped on the X-ray bandwagon, however the work by Arthur Compton demonstrated that electrons and X-rays could exchange energy and momentum, seemingly proving that light again was a particle… but refracts and diffracts like a wave… so what then? Do we call light a particle or a wave? A better theory had to be created; hence the birth of **quantum mechanics**.

Quantum mechanics did not limit its scope to the behaviour of light. It also extended itself to the behaviour of matter as well; predicting that particles, such as protons, electrons and neutrons also have a wave nature. Oddly enough, the wave nature of these particles has been observed. Electron, proton, and neutron diffraction have been observed, proving that these particles behave like waves. This was quite the startling result because one wouldn't expect these particles to experience diffraction anymore then one would expect to observe constructive and destructive interference from the bullet spray of a machine gun.

Problems with the Classical or Wave Theory of Light

1. Wave theory predicts that the energy of any oscillating system could have any value, depending on amplitude, but this effect is not observed in light.

   Ex. the damage caused by an earthquake increases as the amplitude increases. This is expected. However, no matter how many light bulbs you stand under, you will never get a tan, regardless of light bulb brightness. In this case energy transfer is related to frequency, unlike mechanical waves where energy transfer is independent of frequency.

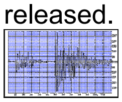
2. Light sometimes exhibits particulate properties such as momentum, implying light should have mass - a property that waves do not have.

3. Electrons, protons and neutrons are particles and shouldn't display any wave characteristics but in fact experience diffraction

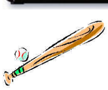
4. All moving charges should create electromagnetic radiation, implying that orbiting electrons should radiate their energy away, consequently slow down (conservation of energy), and crash in to the nucleus. However orbiting electrons do not exhibit these properties.

The Concept of Quantized Energy.

The classical physics idea of energy transfer tells us that the greater the amplitude the great the amount of energy released. Examples

 Earthquakes

a very LOUD guitar amp

 a swinging baseball bat, etc.

However, no one would expect to get sunburn from any of these devices, no matter what the amplitude of each device (WARNING: do not attempt test theory… especially with the bat) For that matter, no one would expect to get sunburn from incandescent light bulbs. But why?

Max Planck:

In 1900, Max Planck suggested that light is made of small discrete energy packets that he called **quanta**. He believed that energy associated with each wavelength of light was stored in these small **quanta**. Therefore instead of a wave, the mechanism for energy transfer in light is by a stream of "little bullets" where each bullet contains the energy associated with the wavelength – intensity or "brightness" is associated with the number of these **quanta** that are delivered over time (similar to the concept of voltage and current whereby the greater the voltage, the greater amount of work that can be done per unit charge, however the quantity of charge is related to the number of charges that flow over time.

Planck believed that the smallest unit of energy (packet) that could be delivered by any given wavelength of light was given by the equation

$$E_\gamma = hf$$

Where E_γ is the energy of a given quantum in Joules (J), h is Planck's constant ($6.626 \times 10^{-34} J \cdot s$) and f is the frequency in Hertz (Hz). Note ($1Hz = s^{-1}$)

since $c = f\lambda$ or $f = \frac{c}{\lambda}$, therefore

$$E_\gamma = \frac{hc}{\lambda}$$

Where E_γ is the energy of a given quantum in Joules (J), h is Planck's constant ($6.626 \times 10^{-34} J \cdot s$), c is the speed of light ($3.0 \times 10^{8} m/s$) and λ is the wavelength in meters (m).

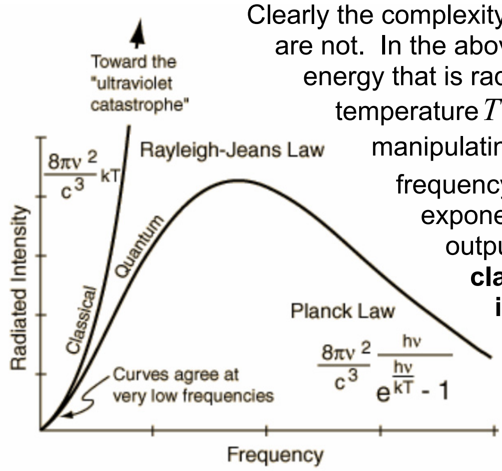
The Ultraviolet Catastrophe

The *coup de grace* for classical mechanics was the so-called "Ultraviolet Catastrophe"

At the beginning of the 20th, the energy signature of a spectrum given off by a **black body** (an ideal body that emits and absorbs all frequencies of electromagnetic energy) could be given by the very complex formula below

$$E(f) = \frac{8\pi f^2}{c^3} kT$$

where E is in Joules (J), f is the frequency in Hz and k is the Boltzmann constant which equals $1.3806503 \times 10^{-23}$ m^2 kg s^{-2} K^{-1} and T is the absolute temperature in Kelvin (K)

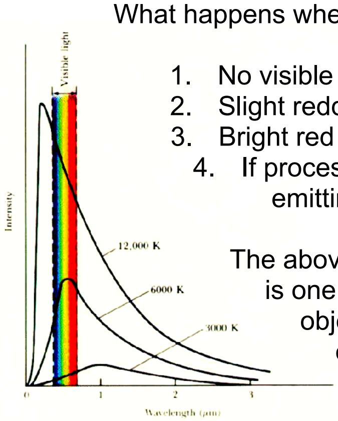


Clearly the complexity of this equation is beyond the scope of this course, but its implications are not. In the above equation, one can see that this formula gives you the amount of energy that is radiated for any given frequency of an electromagnetic wave, at some given temperature T. So this means if you plug in the temperature of your black body and manipulating the f parameter, you will get how much energy is radiated at that frequency. Clearly, as you go higher in frequency, the energy output increases exponentially, implying that as you approach very high frequencies, the energy output approaches infinity! That doesn't happen. Because of this, **the classical mechanical model for electromagnetic radiation was invalidated**… hence the ***catastrophe***.

Applying Planck's concept of the quanta solved the problem, and more accurately predicted the spectra released by a given black body, giving his theory widespread support. **Planck** started a revolution in physics.

The Black Body Equations

What happens when metal is heated? Such as what occurs in a stove element.



1. No visible effect however heat is detected
2. Slight reddish glow appears, more heat detected
3. Bright red glow appears, heat is intensified
4. If process continues unabated, the colour becomes more yellow, eventually turning white emitting intense heat. (Not a normal stove obviously)

The above effect is part of the behaviour black body spectrum. As the temperature rises, there is one wavelength of radiation that becomes very dominant. We often observe it in hot objects (“red hot”). The reason why an object appears to be a certain colour is because one wavelength becomes predominant. This wavelength can be found using the formula

$$\lambda_{\max} = \frac{2.393\times10^{-3}}{T}$$

where $\lambda_{\max}$ is the wavelength in m and T is the temperature in Kelvin (K)

Photoelectric Effect:

The photoelectric effect is further proof that the wave nature of light was an incomplete model; wave theory was unable to properly predict the behaviour of electrons in metal when bombarded with light.

The photoelectric effect is the phenomenon whereby metal can be liberated of an electron by certain frequencies of light but not others. Here is the analogy

Imagine swinging a baseball bat at a very specific speed and being unable to knock a T-ball of its stand. Now imagine getting a bat that is several times heavier and still be unable to knock off that T-ball. However, on the next swing you increase the speed of your swing and the ball goes flying. Furthermore, you reduce the weight of the bat by a factor of ten and you are STILL able to knock off the T-ball. It is counter intuitive to think that only the swing speed should make any difference but not the size of the bat. We would expect that the large bat, swung at the lower speed, should have at least the same effect as the lighter bat swung at the higher speed. This does not happen. Even with a bat that is infinitely heavy, the T-ball will never get liberated. Clearly, this is a ridiculous scenario in our baseball analogy, but is what precisely occurs with the photoelectric effect but instead of *speed* we consider *energy per quantum*.

The concept: every frequency of light has a certain amount of energy associated with each **photon** or **quantum.** If the energy of the photon is too small, the electron in the metal will not be liberated, but the electron will absorb the photon’s energy. This is the reason why metal gets warmer when subjected to certain frequencies of light. If the photon’s energy is sufficient, the electron will be liberated.

Fig.12.11a Lower-energy photons don’t possess enough energy to liberate electrons from a metal surface

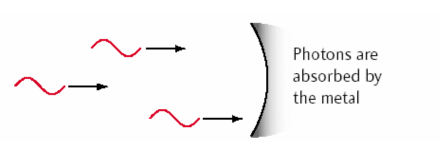
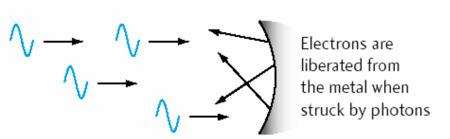


Fig.12.11b If a higher-energy photon hits the metal and is not reflected, it interacts with the electrons in the metal and transfers its energy to an electron. If the energy transferred by the photon is greater than the minimum energy required to evict the electron from the metal, then the electron will be emitted. The electron’s kinetic energy is the energy of the photon minus the energy required to liberate the electron.



Einstein work in this field earned him a Nobel Prize. He set up an apparatus to determine the minimum amount of energy required to liberate an electron from any given metal

How it works. Every electron is bound to an atom by its binding energy. To liberate the electron, the photon must at least deliver the minimum amount of energy to break the electron lose. At this minimum energy level, the electron is free but has no kinetic energy. Einstein called this the **work function**.

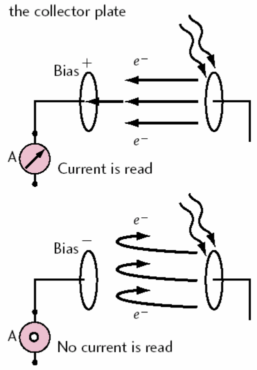


Fig.12.10b The effects of switching the polarity of the collector plate

$E_{photon} = E_{\gamma} = hf_0$ (Planck's equation)

The total energy of photon is split between the liberating energy (**work function** or W_o) and the $E_{k_{max}}$ of the electron

$E_{photon} = E_{k_{max}} + W_o$

$$\therefore hf = \frac{hc}{\lambda} = E_{k_{max}} + W_o$$

Where $E_{k_{max}}$ is the energy of a given quantum in Joules (J), h is Planck's constant ($6.626 \times 10^{-34} J \cdot s$), f is the frequency in Hertz (Hz), and W_o is in Joules (J)

The Electron Volt

Quite often the **work function** (W_o) is measured in **electron volts**. The electron volt is based on the energy of a single electron, expressed in terms of elementary charges.

$$1eV = 1.6 \times 10^{-19} J$$

Example: An electron is accelerated through a potential difference of 5000V. How much energy is imparted to the electron in a) Joules b) Electron-volts.

a) $V = \frac{E}{q}$ (from electrostatics) $E = qV$ $E = 1.6 \times 10^{-19}(5000)$ $E = 8.0 \times 10^{-16} J$	b) $V = \frac{E}{q}$ (from electrostatics) $E = eV$ $E = 1(5000)$ $E = 5000eV$

Example: Find the work function of an unknown metal that begins to experience the photoelectric effect when subjected to light of $\lambda = 510nm$

Given	**RTF**	**Formula**
$\lambda = 510nm$ $= 5.1 \times 10^{-7} m$ $h = 6.626 \times 10^{-34} J \cdot s$	W_0	$\frac{hc}{\lambda} = E_{k_{max}} + W_o$

Solution

$\frac{hc}{\lambda} = E_{k_{max}} + W_o$ but $E_{k_{max}} = 0J$ when the threshold wavelength for the photoelectric effect.

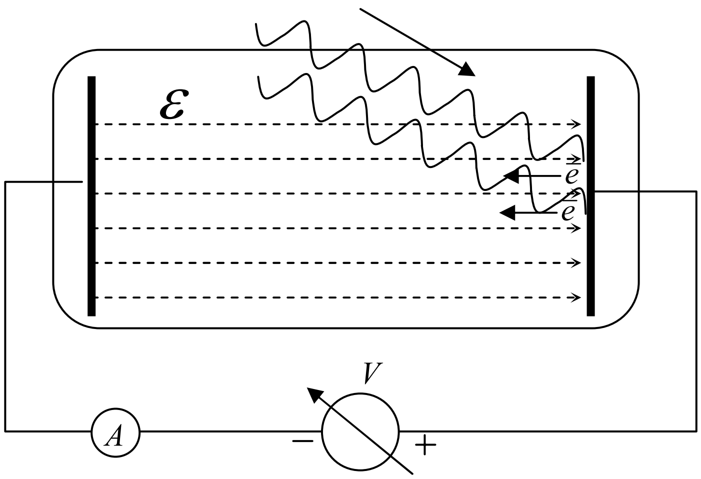
$$\frac{hc}{\lambda} = 0 + W_o$$

$$W_0 = \frac{hc}{\lambda}$$

$$W_0 = \frac{6.626 \times 10^{-34}(3.0 \times 10^8)}{5.1 \times 10^{-7}}$$

$$W_0 = 3.89764 \times 10^{-19} J$$
$$= 2.44eV$$

Stopping Voltage: To determine the kinetic energy of the liberating electron is a rather trivial matter. $E_{photon} = hf = E_{k_{\max}} + W_o$, $E_{k_{\max}}$ is the kinetic energy of the liberated electron. When the electron is liberated by the incident photon, it will have a certain amount kinetic energy or $E_{k_{\max}}$. When light hits the metal, the metal becomes negatively charged. The electrons will then try to move to the opposite electrode or the anode (to the left in this case) and a current will be read in the ammeter in the circuit. By setting up a counter voltage in the circuit, the electrons can be stopped at the anode by converting the electron's kinetic energy to electric potential energy or E_e



$$E_{k_1} + E_{e_1} = E_{k_2} + E_{e_2}$$
$$E_{k_{\max}} + 0 = 0 + qV_{stop}$$
$$hf - W_o = eV_{stop}$$
$$V_{stop} = \frac{hf - W_o}{e}$$
$$\boxed{V_{stop} = \frac{h}{e} f - \frac{W_o}{e}}$$

$$E_{k_1} + E_{e_1} = E_{k_2} + E_{e_2}$$
$$E_{k_{\max}} + 0 = 0 + qV_{stop}$$
$$E_{k_{\max}} = eV_{stop}$$
$$but \quad E_{photon} = E_{k_{\max}} + W_o$$
$$\boxed{E_{photon} = eV_{stop} + W_o}$$

Example: Find the stopping voltage of a metal whose work function is 3.1eV and subjected to light with a wavelength of 100nm.

Photons, Energy, Momentum and the Compton Effect

Arthur Compton experimented heavily with X-rays in the early 1920 - observing the behaviour of the X-ray photon. He noticed something peculiar about the incident X-rays compared to the scattered X-rays. He observed that the scattered X-rays had less energy then the incident X-rays. He conjectured that the energy loss was converted to the kinetic energy of the electrons. If this was the case, then photons must have mass; however, they do not. To get around this problem, Compton used Einstein's idea that energy and matter are interchangeable.

Einstein demonstrated in his special theory of relativity that mass and energy were interchangeable ($E = mc^2$). Therefore rearranging for mass, the "mass-like" value, called the **mass equivalence**, for any given photon is defined as.

$$m = \frac{E_{photon}}{c^2} \qquad m = \frac{hf}{c^2}$$

or

$$m = \frac{hf}{c^2} \quad and \quad c = f\lambda$$

$$m = \frac{h}{c^2}\frac{c}{\lambda}$$

$$m = \frac{h}{c\lambda}$$

Conservation of energy states that

$$E_{photon_1} = E_{k_{electron}} + E_{photon_2}$$

$$E_{x-ray\,i} = \frac{1}{2}m_e v^2 + E_{x-ray\,f}$$

$$E_{x-ray\,i} = \frac{1}{2}m_e v^2 + hf_f$$

where E_{x-ray_i} is the energy of the incident X-ray in Joules, m_e is the mass of the electron in kg, v is the velocity of the emitted electron in m/s, $h = 6.626\times10^{-34} J\cdot s$ (Planck's constant), f_f is the frequency of the reflected X-ray in Hz.

Compton Effect: Compton concluded that if the conservation of energy applies to the interaction between X-rays and electrons, the conservation of momentum must also apply. Therefore:

$$\vec{p}_{x-ray_i} = \vec{p}_{electron} + \vec{p}_{x-ray_2}$$

Compton also needed to find away of expressing the momentum of the X-ray photons by using the **mass equivalence**

$$p = mv$$

but $v = c$ and $m = \frac{h}{c\lambda} \; or = \frac{hf}{c^2}$

$$p = \frac{h}{c\lambda}c \qquad \text{or} \qquad p = \frac{hf}{c^2}c$$

$$p = \frac{h}{\lambda} \qquad\qquad p = \frac{hf}{c}$$

Relating Energy to Momentum

Energy of a photon is given by $E = hf$

and momentum is given by $p = \frac{hf}{c}$ or $hf = pc$

Therefore $\boxed{pc = E}$ or $\boxed{p = \frac{E}{c}}$ where E is in Joules (J), c is the speed of light in m/s , and p is in $N \cdot s$.

(note: momentum of the photon is given in $N \cdot s$, not $kg \cdot m/s$ because photons have no mass and the latter unit would have no meaning)

Example:

An 85-eV x-ray photon collides with an electron. The resultant photon is deflected 60^o form the original line of travel and has a wavelength of 214 nm .

a) What is the momentum of the original photon?
b) What is the momentum of the resultant photon?
c) How much energy was imparted to the electron?
d) How much has the energy calculated in c) above increased the electron's speed?
e) What implications does this speed have for the electron?