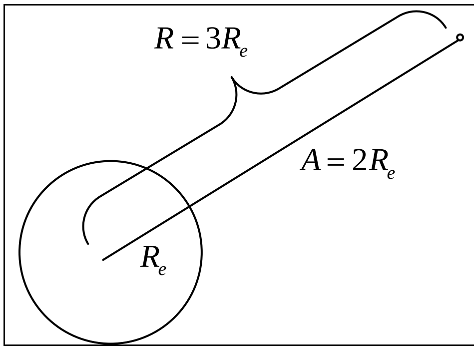
Name:			Thursday, June 23, 2011
Ku: /	App: /	T&I: /	Com:

1. A planet of mass $4.5\times10^{26}kg$ has a radius of $5.3\times10^{6}m$. If a satellite is located 2 radii above the surface determine: [ku:14]

   a) the force of attraction between the satellite and planet if the satellite has a mass of $23486kg$ [3]



$$F_g=\frac{GMm}{R^2}$$
$$=\frac{6.67\times10^{-11}\left(4.5\times10^{26}\right)\left(23486\right)}{\left(3\left(5.3\times10^{6}\right)\right)^2}$$
$$=2789641.83N$$

   b) the period of the satellite and its tangential velocity [5]

$$F_c=F_g$$
$$ma_c=\frac{GMm}{R^2}$$
$$\frac{m4\pi^2R}{T^2}=\frac{GMm}{R^2}$$
$$\frac{4\pi^2R}{T^2}=\frac{GM}{R^2}$$
$$\frac{4\pi^2R^3}{GM}=T^2$$
$$T=\sqrt{\frac{4\pi^2R^3}{GM}}$$
$$T=\sqrt{\frac{4\pi^2\left(3\left(5.3\times10^{6}\right)\right)^3}{6.67\times10^{-11}\left(4.5\times10^{26}\right)}}$$
$$T=2298.83995\ s$$

$$F_c=F_g$$
$$ma_c=\frac{GMm}{R^2}$$
$$\frac{mv^2}{R}=\frac{GMm}{R^2}$$
$$v^2=\frac{GM}{R}$$
$$v=\sqrt{\frac{GM}{R}}$$
$$v=\sqrt{\frac{6.67\times10^{-11}\left(4.5\times10^{26}\right)}{3\left(5.3\times10^{6}\right)}}$$
$$v=43457.8521\ m/s$$

c) the amount of energy required to increase the orbital radius by 20% [4]

$$E_{g_1} + E_{k_1} + E_{Chem_1} = E_{g_2} + E_{k_2} + E_{Chem_2}$$

$$E_{g_1} + E_{k_1} + E_{Chem_1} = E_{g_2} + E_{k_2} + 0$$

$$E_{Chem_1} = E_{g_2} + E_{k_2} - E_{g_1} - E_{k_1}$$

$$E_{Chem_1} = \frac{-GMm}{R_2} + E_{k_2} - \frac{-GMm}{R_1} - E_{k_1}$$

$$E_{Chem_1} = \frac{-GMm}{1.2R} + E_{k_2} + \frac{GMm}{R} - E_{k_1}$$

$$E_{Chem_1} = \frac{-GMm}{1.2R} + \frac{GMm}{2.4R} + \frac{GMm}{R} - \frac{GMm}{2R}$$

$$E_{Chem_1} = \frac{GMm}{R}\left(-\frac{1}{1.2} + \frac{1}{2.4} + 1 - \frac{1}{2}\right)$$

$$E_{Chem_1} = \frac{GMm}{R}\left(-\frac{1}{2.4} + \frac{1}{2}\right)$$

$$E_{Chem_1} = \frac{GMm}{R}\left(\frac{1}{2} - \frac{1}{2.4}\right)$$

$$E_{Chem_1} = \frac{GMm}{R}\left(\frac{2.4}{4.8} - \frac{2}{4.8}\right)$$

$$E_{Chem_1} = \frac{GMm}{R}\left(\frac{0.4}{4.8}\right)$$

$$E_{Chem_1} = \frac{GMm}{R}\left(\frac{1}{12}\right)$$

$$E_{Chem_1} = \frac{\left(6.67\times10^{-11}\right)\left(4.5\times10^{26}\right)\left(23486\right)}{\left(3\left(5.3\times10^{6}\right)\right)}\left(\frac{1}{12}\right)$$

$$E_{Chem_1} = 3.69627542\times10^{12}\,J$$

Let $R_1 = R$ and $R_2 = 1.2R$

Aside E_{k_1}

$$F_{c_1} = F_{g_1}$$

$$ma_{c_1} = \frac{GMm}{R^2{}_1}$$

$$\frac{mv_1^{\,2}}{R_1} = \frac{GMm}{R^2{}_1}$$

$$mv_1^{\,2} = \frac{GMm}{R_1}$$

(but $R_1 = R$)

$$\frac{mv_1^{\,2}}{2} = \frac{GMm}{2R_1}$$

$$\frac{mv_1^{\,2}}{2} = \frac{GMm}{2R}$$

$$E_{k_1} = \frac{GMm}{2R}$$

Aside E_{k_2}

By extension

$$E_{k_1} = \frac{GMm}{2R_2}$$

$$E_{k_1} = \frac{GMm}{2(1.2R)}$$

$$E_{k_1} = \frac{GMm}{2.4R}$$

d) the acceleration due to gravity on the surface of the planet. [2]

$$F_g = \frac{GMm}{R^2}$$

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2}$$

$$g = \frac{6.67\times10^{-11}\left(4.5\times10^{26}\right)}{\left(5.3\times10^{6}\right)^2}$$

$$g = 1069.01032\,m/s^2$$

2. If the length of the day on the planet is 15 Earth hours, determine the amount of energy lost if one were to land at a point on the planet with a latitude of 22° from the position of 3 radii above the surface. Solve algebraically first. (assume the mass of your space craft is $987234kg$) [ku:10]

$$\Delta E_T = E_{T_2} - E_{T_1}$$
$$= \left(E_{g_2} + E_{k_2}\right) - \left(E_{g_1} + E_{k_1}\right)$$
$$= \left(\frac{-GMm}{R} + E_{k_2}\right) - \left(\frac{-GMm}{4R} + \frac{GMm}{8R}\right)$$
$$= \left(\frac{-GMm}{R} + E_{k_2}\right) - \left(-\frac{GMm}{8R}\right)$$
$$= \frac{-GMm}{R} + E_{k_2} + \frac{GMm}{8R}$$
$$= E_{k_2} = \frac{-8GMm}{8R} + \frac{GMm}{8R}$$
$$= E_{k_2} - \frac{7GMm}{8R}$$
$$= 2m\left(\frac{\pi R\cos\theta}{T}\right)^2 - \frac{7GMm}{8R}$$
$$= m\left[2\left(\frac{\pi R\cos\theta}{T}\right)^2 - \frac{7GM}{8R}\right]$$
$$= 987234\left[2\left(\frac{\pi\left(5.3\times10^{6}\right)\cos(22)}{54000}\right)^2 - \frac{7\left(6.67\times10^{-11}\right)\left(4.5\times10^{26}\right)}{8\left(5.3\times10^{6}\right)}\right]$$
$$= -4.87810963\times10^{15}\,J$$

Let $R_1 = 4R$ *and* $R_2 = R$

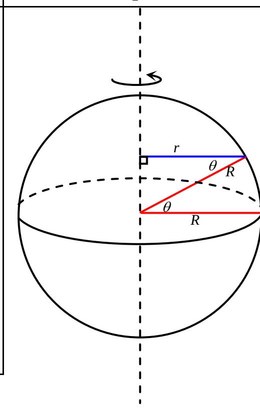
Aside E_{k_1}

$$F_{c_1} = F_{g_1}$$
$$ma_{c_1} = \frac{GMm}{R^2_1}$$
$$\frac{mv_1^2}{R_1} = \frac{GMm}{R^2_1}$$
$$mv_1^2 = \frac{GMm}{R_1}$$

(but $R_1 = R$)

$$\frac{mv_1^2}{2} = \frac{GMm}{2R_1}$$
$$\frac{mv_1^2}{2} = \frac{GMm}{2(4R)}$$
$$E_{k_1} = \frac{GMm}{8R}$$

Aside E_{k_2}



$$\cos\theta = \frac{r}{R}$$
$$r = R\cos\theta$$

Tangential velocity of the planet at landing site

$$v_2 = \frac{C}{T}$$
$$v_2 = \frac{2\pi r}{T}$$
$$v_2 = \frac{2\pi R\cos\theta}{T}$$

Finding E_{k_2}

$$E_{k_2} = \frac{1}{2}mv_2^2$$
$$E_{k_2} = \frac{1}{2}m\left(\frac{2\pi R\cos\theta}{T}\right)^2$$
$$E_{k_2} = \frac{4}{2}m\left(\frac{\pi R\cos\theta}{T}\right)^2$$
$$E_{k_2} = 2m\left(\frac{\pi R\cos\theta}{T}\right)^2$$

3. Two point charges of mass $1.50g$ and $2.50g$ have a charge of $-2.00\times10^{-3}C$ and $4.50\times10^{-2}C$. If the point charges are located initially $10.0m$ away from each other determine their respective speeds when the reach a separation distance of $1.00m$ assuming they both start from rest. [ku:10]

<u>From conservation of Energy</u>

$$E_{e_1}+E_{k_{A_1}}+E_{k_{B_1}}=E_{e_2}+E_{k_{A_2}}+E_{k_{B_2}}$$

$$E_{e_1}+0+0=E_{e_2}+E_{k_{A_2}}+E_{k_{B_2}}$$

$$E_{e_1}-E_{e_2}=E_{k_{A_2}}+E_{k_{B_2}}$$

$$E_{k_{A_2}}+E_{k_{B_2}}=E_{e_1}-E_{e_2}$$

$$E_{k_{A_2}}+E_{k_{B_2}}=\frac{kq_Aq_B}{d_1}-\frac{kq_Aq_B}{d_2}$$

$$E_{k_{A_2}}+E_{k_{B_2}}=kq_Aq_B\left(\frac{1}{d_1}-\frac{1}{d_2}\right)$$

$$\frac{1}{2}m_Av_A^2+\frac{1}{2}m_Bv_B^2=kq_Aq_B\left(\frac{1}{d_1}-\frac{1}{d_2}\right)$$

$$m_Av_A^2+m_Bv_B^2=2kq_Aq_B\left(\frac{1}{d_1}-\frac{1}{d_2}\right)$$

(equation 1)

<u>From conservation of momentum</u>

$$\vec{p}_{A_1}+\vec{p}_{B_1}=\vec{p}_{A_2}+\vec{p}_{B_2}$$

$$\vec{p}_{A_1}+\vec{p}_{B_1}=\vec{p}_{A_2}+\vec{p}_{B_2}\quad\text{(assuming charge A is moving left)}$$

$$0+0=-m_Av_A+m_Bv_B$$

$$m_Av_A=m_Bv_B$$

$$v_A=\frac{m_Bv_B}{m_A}$$

(equation 2)

<u>Sub eq'n (2) into (1)</u>

$$m_A\left(\frac{m_Bv_B}{m_A}\right)^2+m_Bv_B^2=2kq_Aq_B\left(\frac{1}{d_1}-\frac{1}{d_2}\right)$$

$$m_A\frac{m_B^2v_B^2}{m_A^2}+m_Bv_B^2=2kq_Aq_B\left(\frac{1}{d_1}-\frac{1}{d_2}\right)$$

$$\frac{m_B^2v_B^2}{m_A}+m_Bv_B^2=2kq_Aq_B\left(\frac{1}{d_1}-\frac{1}{d_2}\right)$$

$$\left(\frac{m_B^2}{m_A}+m_B\right)v_B^2=2kq_Aq_B\left(\frac{1}{d_1}-\frac{1}{d_2}\right)$$

$$m_B\left(\frac{m_B}{m_A}+1\right)v_B^2=2kq_Aq_B\left(\frac{1}{d_1}-\frac{1}{d_2}\right)$$

$$v_B=\sqrt{\frac{2kq_Aq_B}{m_B\left(\frac{m_B}{m_A}+1\right)}\times\left(\frac{1}{d_1}-\frac{1}{d_2}\right)}$$

$$v_B=\sqrt{\frac{2\left(9.09\times10^{9}\right)\left(-2.00\times10^{-3}\right)\left(4.5\times10^{-2}\right)}{\left(0.00250\right)\left(\frac{0.00250}{0.00150}+1\right)}\times\left(\frac{1}{10}-\frac{1}{1}\right)}$$

$$v_B=14862.2677m/s$$

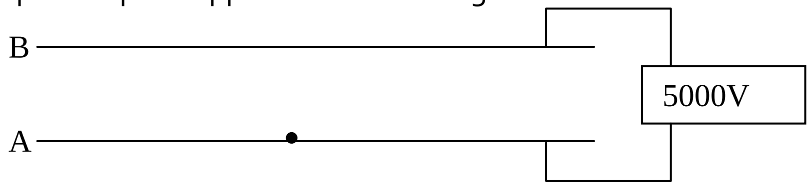
Find v_A

$$v_A=\frac{m_Bv_B}{m_A}$$

$$v_A=\frac{0.00250\left(14862.2677\right)}{0.00150}$$

$$v_A=24770.4462m/s$$

4. A parallel plate apparatus is set configured as demonstrated below. [ku:8]



The small particle has a mass of $1.3\times10^{-4}kg$ and has a charge of $-5.8\times10^{-6}C$. If the plates are separated by 12cm determine

a) the relative voltage of plates A and B (2)

Since the particle is negatively charged, the bottom plate must be negatively charged and the top plate must be positively charged in order for the particle to move upward.
Therefore $V_A = 0V$ and $V_B = +5000V$

b) the speed at which the particle arrives at plate B (note the change of Eg is NOT insignificant and must be included... and of course... solve algebraically. (6)

Using conservation of energy

$$E_{e_A} + E_{k_A} + E_{g_A} = E_{e_B} + E_{k_B} + E_{g_B}$$

$$E_{e_A} + 0 + 0 = E_{e_B} + E_{k_B} + E_{g_B}$$

$$E_{e_B} + E_{k_B} + E_{g_B} = E_{e_A}$$

$$E_{k_B} = E_{e_A} - E_{e_B} - E_{g_B}$$

$$\frac{1}{2}mv_B{}^2 = q_tV_A - q_tV_B - mgh_B$$

$$\frac{1}{2}mv_B{}^2 = q_t\left(V_A - V_B\right) - mgh_B$$

$$v_B{}^2 = \frac{2q_t}{m}\left(V_A - V_B\right) - \frac{mgh_B}{m}$$

$$v_B = \sqrt{\frac{2q_t}{m}\left(V_A - V_B\right) - gh_B}$$

$$v_B = \sqrt{\frac{2\left(-5.8\times10^{-6}\right)}{\left(1.3\times10^{-4}\right)}\left(0-5000\right) - \left(9.8\right)\left(0.12\right)}$$

$$v_B = 21.094498m/s$$